# Testing *Low Scale* Leptonic Unitarity Violation

Chee Sheng Fong
University of São Paulo, Brazil

**I**ceCube **P**article **A**strophysics Symposium
University of Wisconsin-Madison
May 9, 2017

*Based on hep-ph/1609.08623 [JHEP 1702 (2017) 114]*
*with H. Minakata (Yachay Tech, Ecuador) & H. Nunokawa (PUC, Brazil)*

# Outline

- Motivations
- Low scale unitarity violation in neutrino oscillations
- Experimental sensitivity: JUNO
- Matter effects
- Remarks

# Motivations

# Neutrinos are ***Special***

- Like other SM fermions, they are massive (at least 2 of 3)
- Unlike the other SM fermions, they are the only electric charge neutral ones!  This means only they can be Majorana particles!

# Neutrinos are *Special*

- Like other SM fermions, they are massive (at least 2 of 3)
- Unlike the other SM fermions, they are the only electric charge neutral ones! This means only they can be Majorana particles!
- According to Gell-Mann totalitarian principle: “everything not forbidden is compulsory.” So they must be Majorana …

# Neutrinos are (portal to) *New Physics*

- If Majorana, lepton number is violated → ***Leptogenesis***!

# Neutrinos are (portal to) *New Physics*

- If Majorana, lepton number is violated → ***Leptogenesis***!
- Neutrino masses are ***New Physics*** either way

$$m_D \bar{\nu}_L \nu_R \quad \to y_\nu \bar{\ell}_L \tilde{H} \nu_R \qquad \ell_L = (\nu_L, e_L)$$

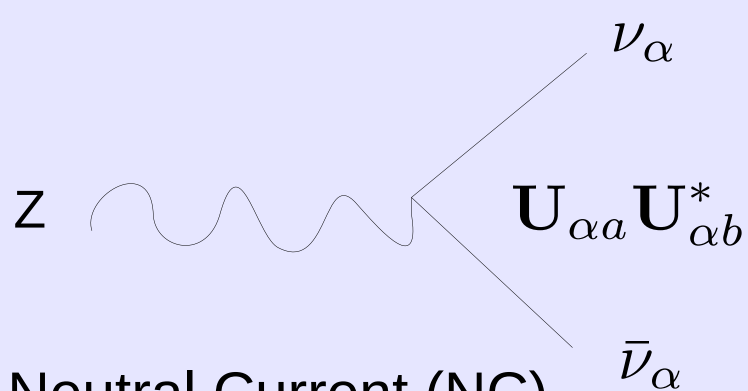
$$m_M \bar{\nu}_L \nu_L^c \quad \to \frac{1}{\Lambda} (\bar{\ell}_L \tilde{H})(\ell_L^c \tilde{H}) \qquad H = (h^+, h^0)$$

- They are *active* – participate in weak interactions!
- SU(2) gauge invariance further dictates that new physics couples to charged leptons → **lepton nonuniversality?**
- If there exists additional neutral fermions which do not participate in weak interaction i.e. *sterile* neutrinos, it is possible that they mix with the active neutrinos

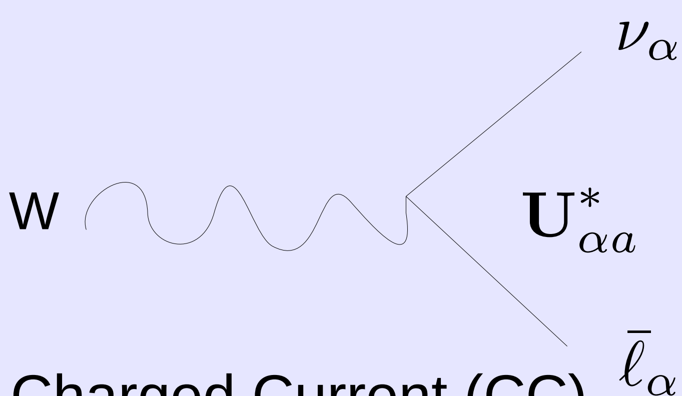
→ "Leptonic *Unitarity Violation* (UV)"

# *High* and Low scale UV

Deviations in weak interactions




Neutral Current (NC)




Charged Current (CC)

$$\nu_\alpha = \mathbf{U}_{\alpha 1}\nu_1 + \mathbf{U}_{\alpha 2}\nu_2 + \mathbf{U}_{\alpha 3}\nu_3 + \mathbf{U}_{\alpha 4}\nu_4 + ...$$

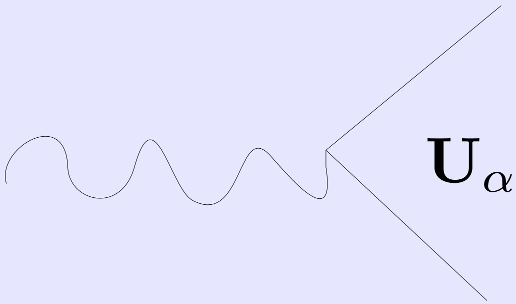
$m_{4,5,...} \gg \mathrm{MeV}$ *Not kinematically* accessible

$$|\mathrm{Amp}|^2 \sim \sum_{a=1}^{3} \mathbf{U}_{\alpha a}\mathbf{U}^*_{\beta a} \neq \delta_{\alpha\beta}$$ Or measurable mass effects

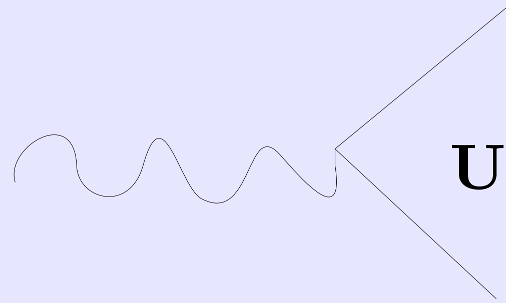
*High* scale UV
→ Lepton universality violation
→ Zero distance flavor transition in neutrino oscillation

# High and *Low* scale UV

Deviations in weak interactions




Neutral Current (NC)




Charged Current (CC)

$$\nu_\alpha = \mathbf{U}_{\alpha 1}\nu_1 + \mathbf{U}_{\alpha 2}\nu_2 + \mathbf{U}_{\alpha 3}\nu_3 + \mathbf{U}_{\alpha 4}\nu_4 + \dots$$

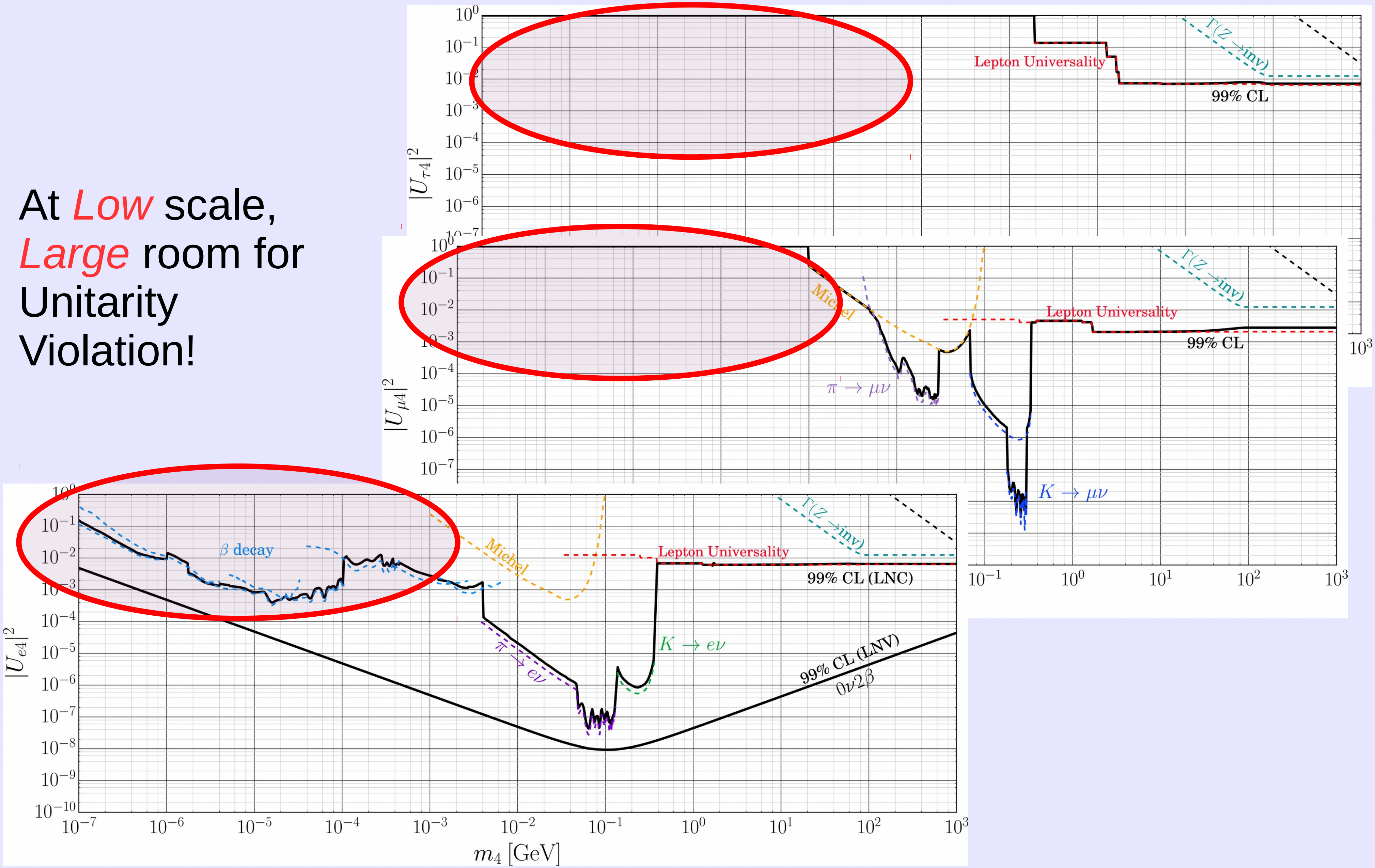
$m_{4,5,\dots} \lesssim \mathrm{MeV}$ *Kinematically* accessible

$$|\mathrm{Amp}|^2 \sim \sum_{a=1}^{\mathrm{all}} \mathbf{U}_{\alpha a}\mathbf{U}^*_{\beta a} = \delta_{\alpha\beta}$$ And negligible mass effects

*Low* scale UV
→ *No* lepton universality violation
→ *No* zero distance flavor transition in neutrino oscillation

# Constraints on 3+1 [de Gouvêa & Kobach (2016)]

At *Low* scale,
*Large* room for
Unitarity
Violation!



# Low scale UV in $\nu$ oscillations

# 3 active + N sterile model

- Neutrino "flavor" eigenstates are related to mass eigenstates through $\nu_\zeta = \mathbf{U}_{\zeta a}\nu_a$

$$\mathbf{U} = \begin{bmatrix} U_{3\times 3} & W_{3\times N} \\ Z_{N\times 3} & V_{N\times N} \end{bmatrix} \qquad \mathbf{U}^\dagger\mathbf{U} = \mathbf{U}\mathbf{U}^\dagger = \mathbf{1}$$

$$\zeta = e, \mu, \tau, s_1, s_2, ..., s_N, \quad a = 1, 2, 3, 4, ..., 3+N$$

active sterile mass eigenstates

- Relevant unitarity relations

$$\sum_i U_{\alpha i} U^*_{\beta i} + \sum_I W_{\alpha I} W^*_{\beta I} = \delta_{\alpha\beta}$$

$$\alpha, \beta, ... = e, \mu, \tau, \quad i, j, ... = 1, 2, 3, \quad I, J, ... = 4, 5, ..., 3+N$$

$$W \equiv \begin{bmatrix} W_{e\,4} & W_{e\,5} & ... & W_{e\,3+N} \\ W_{\mu\,4} & W_{\mu\,5} & ... & W_{\mu\,3+N} \\ W_{\tau\,4} & W_{\tau\,5} & ... & W_{\tau\,3+N} \end{bmatrix}$$

# Low scale UV: no zero distance

- Neutrino oscillation probability in vacuum

$$P(\nu_\beta \to \nu_\alpha) = \left| \sum_a \mathbf{U}_{\alpha a} \mathbf{U}^*_{\beta a} \; e^{-i \frac{m_a^2 x}{2E}} \right|^2$$

- No zero distance effect

$$P(\nu_\beta \to \nu_\alpha)(x=0) = \left| \sum_i U_{\alpha i} U^*_{\beta i} + \underbrace{\sum_I W_{\alpha I} W^*_{\beta I}} \right|^2 = \delta_{\alpha\beta}$$

Sterile states *kinematically* accessible

# Low scale UV: no zero distance

- Neutrino oscillation probability in vacuum

$$P(\nu_\beta \to \nu_\alpha) = \left| \sum_a \mathbf{U}_{\alpha a} \mathbf{U}^*_{\beta a} \; e^{-i\frac{m_a^2 x}{2E}} \right|^2$$

- No zero distance effect

$$P(\nu_\beta \to \nu_\alpha)(x=0) = \left| \sum_i U_{\alpha i} U^*_{\beta i} + \underbrace{\sum_I W_{\alpha I} W^*_{\beta I}} \right|^2 = \delta_{\alpha\beta}$$

Sterile states *kinematically* accessible

- Can we make the expressions ***insensitive*** to details of the *sterile mass spectra & mixing?*

# Low scale UV: ***partial decoherence***

- *Decoherence* occurs when the variation in phase due to spatial and/or energy resolution is greater than 2π

$$\left|\delta\left(\frac{\Delta m^2_{ab} x}{2E}\right)\right| = \left|\frac{\Delta m^2_{ab}}{2E}\delta x - \frac{\Delta m^2_{ab} x}{2E^2}\delta E\right| \gtrsim 2\pi$$

  - Spatial resolution

$$\delta x \gtrsim \frac{4\pi E}{|\Delta m^2_{ab}|}$$

  - Energy resolution

$$\delta E \gtrsim \frac{4\pi E^2}{|\Delta m^2_{ab}| x}$$

# Low scale UV: *partial* decoherence

- *Decoherence* occurs when the variation in phase due to spatial and/or energy resolution is greater than 2π

$$\left|\delta\left(\frac{\Delta m^2_{ab}x}{2E}\right)\right| = \left|\frac{\Delta m^2_{ab}}{2E}\delta x - \frac{\Delta m^2_{ab}x}{2E^2}\delta E\right| \gtrsim 2\pi$$

  - Spatial resolution

$$\delta x \gtrsim \frac{4\pi E}{|\Delta m^2_{ab}|} \qquad |\Delta m^2_{Ja}| \gtrsim 1.2\ \mathrm{eV}^2 \left(\frac{x/x_{\mathrm{prod}}}{5\times 10^3}\right) \quad \text{(for reactor)}$$

  - Energy resolution

$$\delta E \gtrsim \frac{4\pi E^2}{|\Delta m^2_{ab}|x} \quad |\Delta m^2_{Ja}| \gtrsim 7.9\times 10^{-3}\mathrm{eV}^2 \left(\frac{\delta E/E}{0.03}\right)^{-1} \quad \text{(for reactor)}$$

Most experimental settings $\frac{\Delta m^2_{21}x}{4E} \sim 1$ or $\frac{|\Delta m^2_{31}|x}{4E} \sim 1$

# Low scale UV: *partial* decoherence

- *Decoherence* occurs when the variation in phase due to spatial and/or energy resolution is greater than 2π

$$\left|\delta\left(\frac{\Delta m^2_{ab}x}{2E}\right)\right| = \left|\frac{\Delta m^2_{ab}}{2E}\delta x - \frac{\Delta m^2_{ab}x}{2E^2}\delta E\right| \gtrsim 2\pi$$

  - Spatial resolution

$$\delta x \gtrsim \frac{4\pi E}{|\Delta m^2_{ab}|} \qquad |\Delta m^2_{Ja}| \gtrsim 1.2\ \mathrm{eV}^2 \left(\frac{x/x_{\mathrm{prod}}}{5\times 10^3}\right) \quad \text{(for reactor)}$$

  - Energy resolution

More efficient to cause partial decoherence

$$\delta E \gtrsim \frac{4\pi E^2}{|\Delta m^2_{ab}|x} \qquad |\Delta m^2_{Ja}| \gtrsim 7.9\times 10^{-3}\mathrm{eV}^2 \left(\frac{\delta E/E}{0.03}\right)^{-1} \quad \text{(for reactor)}$$

Most experimental settings $\quad \dfrac{\Delta m^2_{21}x}{4E} \sim 1 \quad \text{or} \quad \dfrac{|\Delta m^2_{31}|x}{4E} \sim 1$

# Low scale UV: *partial* decoherence

- For the mass range

$$0.1\ \mathrm{eV}^2 \lesssim m_J^2 \lesssim 1\ \mathrm{MeV}^2$$

Energy of reactor neutrinos

we can average out fast oscillations, assuming no accidental degeneracy i.e. $|\Delta m^2_{JK}| \gg |\Delta m^2_{31}|$

$$\left\langle \sin\left(\frac{\Delta m^2_{Ji} x}{2E}\right)\right\rangle \approx \left\langle \sin\left(\frac{\Delta m^2_{JK} x}{2E}\right)\right\rangle \approx 0$$

$$\left\langle \cos\left(\frac{\Delta m^2_{Ji} x}{2E}\right)\right\rangle \approx \left\langle \cos\left(\frac{\Delta m^2_{JK} x}{2E}\right)\right\rangle \approx 0$$

# "Model independent"

- With partial decoherence, we have

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2$$

Leaking term

$$-2 \sum_{j \neq k} \mathrm{Re}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j)x$$

**"Model independent"**

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=4}^{3+N} |W_{\alpha J}|^2 |W_{\beta J}|^2$$

$U_{\alpha i}$ : generic complex matrix (nonunitary)

$C_{\alpha\beta}$ : real positive term

# “Model independent”

- With partial decoherence, we have

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2$$

Leaking term

$$-2 \sum_{j \neq k} \mathrm{Re}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j)x$$

“Model independent”

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=4}^{3+N} |W_{\alpha J}|^2 |W_{\beta J}|^2$$

Need high precision experiments!

$U_{\alpha i}$ : generic complex matrix (nonunitary)

$C_{\alpha\beta}$ : real positive term

} *Not independent*

# 3+N unitarity constraints

- From the identity

$$\begin{aligned}\mathcal{C}_{\alpha\beta} &= \left(\sum_{I=4}^{3+N}|W_{\alpha I}|^2\right)\left(\sum_{J=4}^{3+N}|W_{\beta J}|^2\right) - \sum_{I\neq J}|W_{\alpha I}W_{\beta J}|^2 \\ &= \left(1-\sum_{i=1}^{3}|U_{\alpha i}|^2\right)\left(1-\sum_{i=1}^{3}|U_{\beta i}|^2\right) - \sum_{I\neq J}|W_{\alpha I}W_{\beta J}|^2\end{aligned}$$

- we derive

$$\sum_{I\neq J}|W_{\alpha I}W_{\beta J}|^2 = 0 \quad\Longrightarrow\quad \mathcal{C}_{\alpha\beta}^{\max} = \left(1-\sum_{i=1}|U_{\alpha i}|^2\right)\left(1-\sum_{i=1}|U_{\beta i}|^2\right)$$

$$W_{\alpha I} = v,\; W_{\beta J} = w \quad\Longrightarrow\quad \mathcal{C}_{\alpha\beta}^{\min} = \frac{1}{N}\left(1-\sum_{i=1}|U_{\alpha i}|^2\right)\left(1-\sum_{i=1}|U_{\beta i}|^2\right)$$

N dependent!

# Probing N

- For $\beta = \alpha$, the 3+N constraints are

$$\frac{1}{N}\left(1-\sum_{i=1}^{3}|U_{\alpha i}|^2\right)^2 \leq \mathcal{C}_{\alpha\alpha} \leq \left(1-\sum_{i=1}^{3}|U_{\alpha i}|^2\right)^2$$

- Rearranging, we have

$$\sqrt{\mathcal{C}_{\alpha\alpha}} \leq (1-\sum_{i=1}^{3}|U_{\alpha i}|^2) \leq \sqrt{N\mathcal{C}_{\alpha\alpha}}$$

- Suppose the data show

$$(1-\sum_{i=1}^{3}|U_{\alpha i}|^2) = \sqrt{M\mathcal{C}_{\alpha\alpha}}$$

**N < M is excluded within our framework**

# Strategy of testing low scale UV

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left|\sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j}\right|^2$$
$$-2\sum_{j\neq k} \mathrm{Re}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}\right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$
$$-\sum_{j\neq k} \mathrm{Im}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}\right) \sin(\Delta_k - \Delta_j)x$$

(A) Nonunitary $U_{\alpha i}$

(B) Unitary $U_{\alpha i}$ where $\mathcal{C}_{\alpha\beta} = 0$

(1) Fit shows a (no) discrepancy between (A) and (B)

→ indication of a (no) UV

# Strategy of testing low scale UV

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U_{\beta j}^* \right|^2$$
$$-2 \sum_{j \neq k} \mathrm{Re}\left( U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j) x}{2}$$
$$- \sum_{j \neq k} \mathrm{Im}\left( U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k} \right) \sin(\Delta_k - \Delta_j) x$$

(A) Nonunitary $U_{\alpha i}$

(B) Unitary $U_{\alpha i}$ where $\mathcal{C}_{\alpha\beta} = 0$

(1) Fit shows a (no) discrepancy between (A) and (B)

→ indication of a (no) UV

(2) Fit prefers nonzero $\mathcal{C}_{\alpha\beta}$ → hint of *low scale* UV

# Strategy of testing low scale UV

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2$$
$$-2 \sum_{j \neq k} \mathrm{Re}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$
$$- \sum_{j \neq k} \mathrm{Im}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j)x$$

(A) Nonunitary $U_{\alpha i}$

(B) Unitary $U_{\alpha i}$ where $\mathcal{C}_{\alpha\beta} = 0$

(1) Fit shows a (no) discrepancy between (A) and (B)

→ indication of a (no) UV

(2) Fit prefers nonzero $\mathcal{C}_{\alpha\beta}$ → hint of *low scale* UV

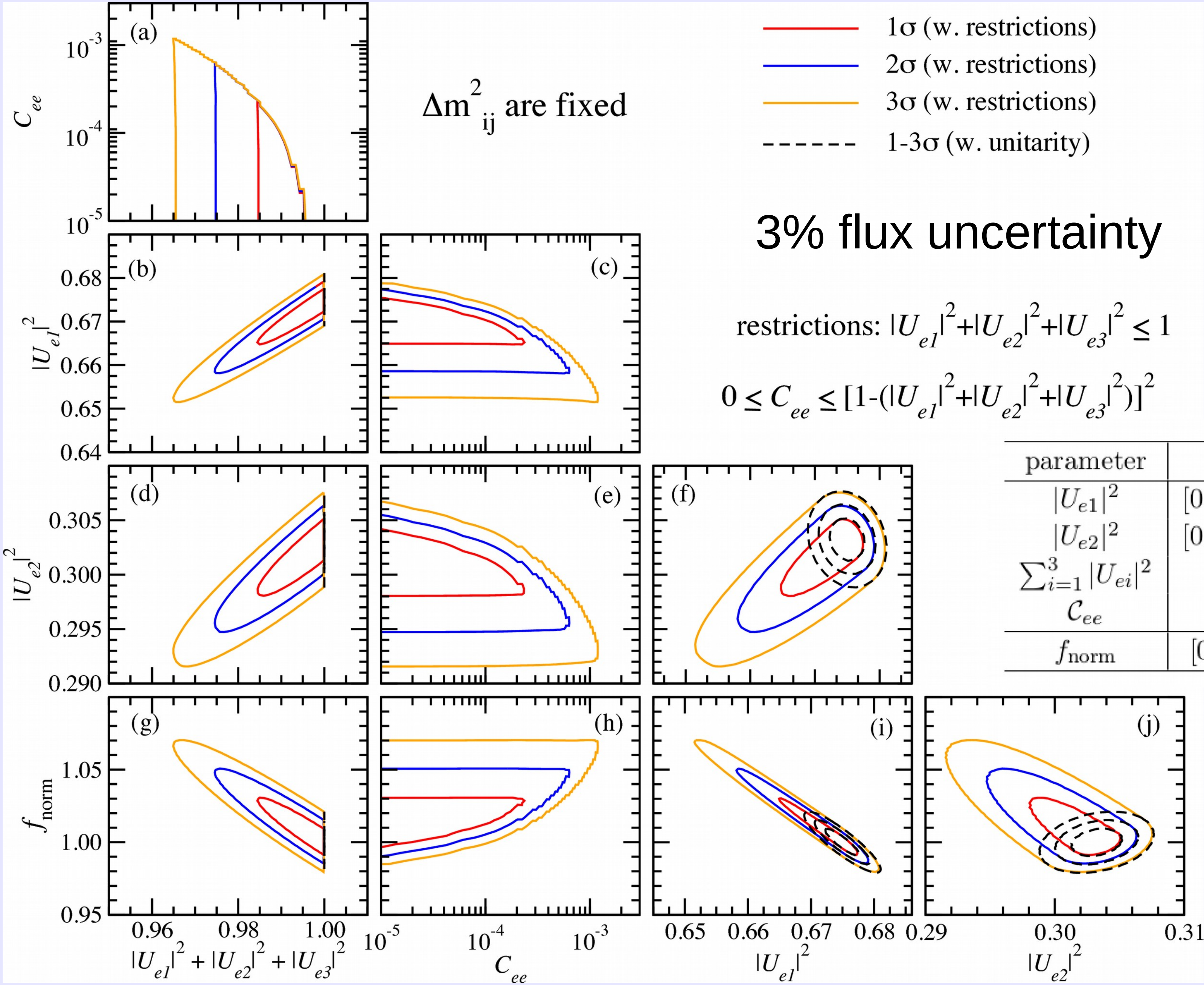
(3) Fit prefers a nonzero $\mathcal{C}_{\alpha\beta}$ but outside “3+N constraints”

→ likely a *low scale* UV but

- Partial decoherence conditions are not fulfilled
- Cannot be described by our 3+N model

# Experimental sensitivity: JUNO

# 5-year JUNO for $\bar{\nu}_e \rightarrow \bar{\nu}_e$




3% flux uncertainty

| parameter | $1\sigma$ range | $3\sigma$ range |
|---|---|---|
| $\|U_{e1}\|^2$ | [0.668, 0.676] | [0.654, 0.680] |
| $\|U_{e2}\|^2$ | [0.299, 0.304] | [0.293, 0.307] |
| $\sum_{i=1}^{3}\|U_{ei}\|^2$ | [0.989, 1] | [0.968, 1] |
| $\mathcal{C}_{ee}$ | $[0, 10^{-4}]$ | $[0, 10^{-3}]$ |
| $f_{\rm norm}$ | [0.994, 1.02] | [0.983, 1.063] |

# Matter effects

# Matter effects

- Introduce matter effects as *perturbations*

$$\text{CC}: A = 2\sqrt{2}G_F N_e E, \quad \text{NC}: B = \sqrt{2}G_F N_n E = \frac{1}{2}\left(\frac{N_n}{N_e}\right) A$$

$$|A| \ll |\Delta m^2_{31}|$$

- No assumption on size of UV besides |W| < 1
- “Model-independent” for $0.1\ \text{eV}^2 \lesssim m_J^2 \lesssim 1\ \text{MeV}^2$ ?

# Matter effects

- Introduce matter effects as *perturbations*

$$\mathrm{CC}: A = 2\sqrt{2} G_F N_e E, \quad \mathrm{NC}: B = \sqrt{2} G_F N_n E = \frac{1}{2}\left(\frac{N_n}{N_e}\right) A$$

$$|A| \ll |\Delta m^2_{31}|$$

- No assumption on size of UV besides |W| < 1
- "Model-independent" for $0.1\ \mathrm{eV}^2 \lesssim m_J^2 \lesssim 1\ \mathrm{MeV}^2$ ?

$$\frac{|\Delta_A|}{(\Delta_J - \Delta_k)} = \frac{|A|}{\Delta m^2_{Jk}} \ll |W|^2$$

Additional requirement

$$\frac{|A|}{\Delta m^2_{Jk}} = 2.13 \times 10^{-3} \left(\frac{0.1\mathrm{eV}^2}{\Delta m^2_{Jk}}\right) \left(\frac{\rho}{2.8\,\mathrm{g/cm}^3}\right) \left(\frac{E}{1\ \mathrm{GeV}}\right)$$

- Generically fulfill unless very small W

$$W^2 \sim 10^{-3} (W^2 \lesssim 10^{-n}) \implies \Delta m^2_{Jk} \simeq m_J^2 \gtrsim 1\,\mathrm{eV}^2 (10^{(n-3)}\ \mathrm{eV}^2)$$

# Remarks

- Neutrinos are (portal to) *New Physics*
- If ***sterile*** neutrinos mix with ***active*** neutrinos → leptonic UV
- *Low* scale UV preserves lepton universality → *Large* room for UV; can be tested in neutrino oscillations
- Partial decoherence allows "model-independent" test

  $0.1\ \mathrm{eV}^2 \lesssim m_J^2 \lesssim 1\ \mathrm{MeV}^2$

- With small matter effects, remains "model-independent"
- Situation with strong matter effects (e.g. IceCube with neutrinos passing through the Earth, DUNE) under study

# Remarks

Thanks for your attention

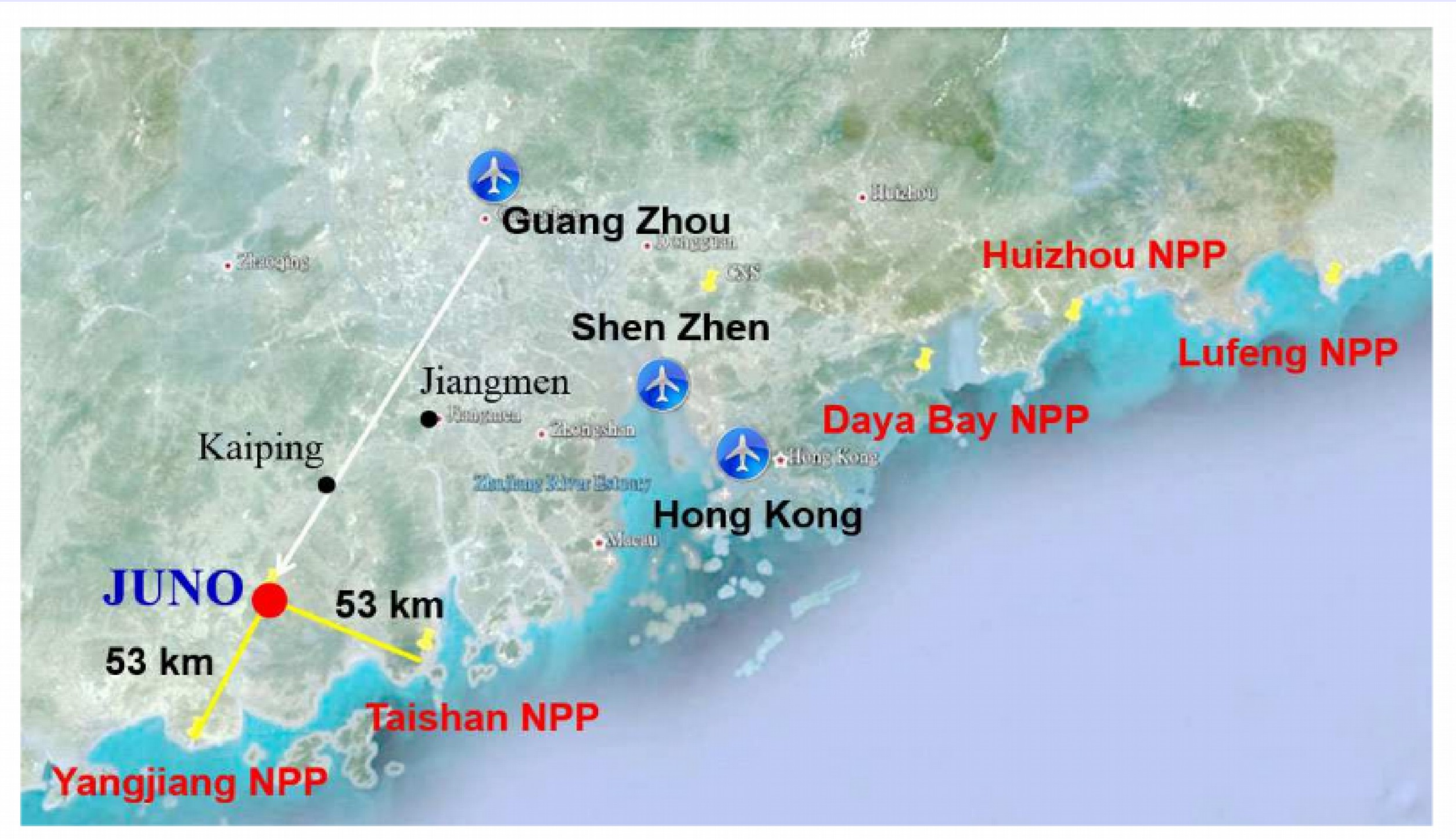
- Neutrinos are (portal to) *New Physics*
- If *sterile* neutrinos mix with *active* neutrinos → leptonic UV
- *Low* scale UV preserves lepton universality → *Large* room for UV; can be tested in neutrino oscillations
- Partial decoherence allows "model-independent" test

  $$0.1\ \mathrm{eV}^2 \lesssim m_J^2 \lesssim 1\ \mathrm{MeV}^2$$

- With small matter effects, remains "model-independent"
- Situation with strong matter effects (e.g. IceCube with neutrinos passing through the Earth, DUNE) under study

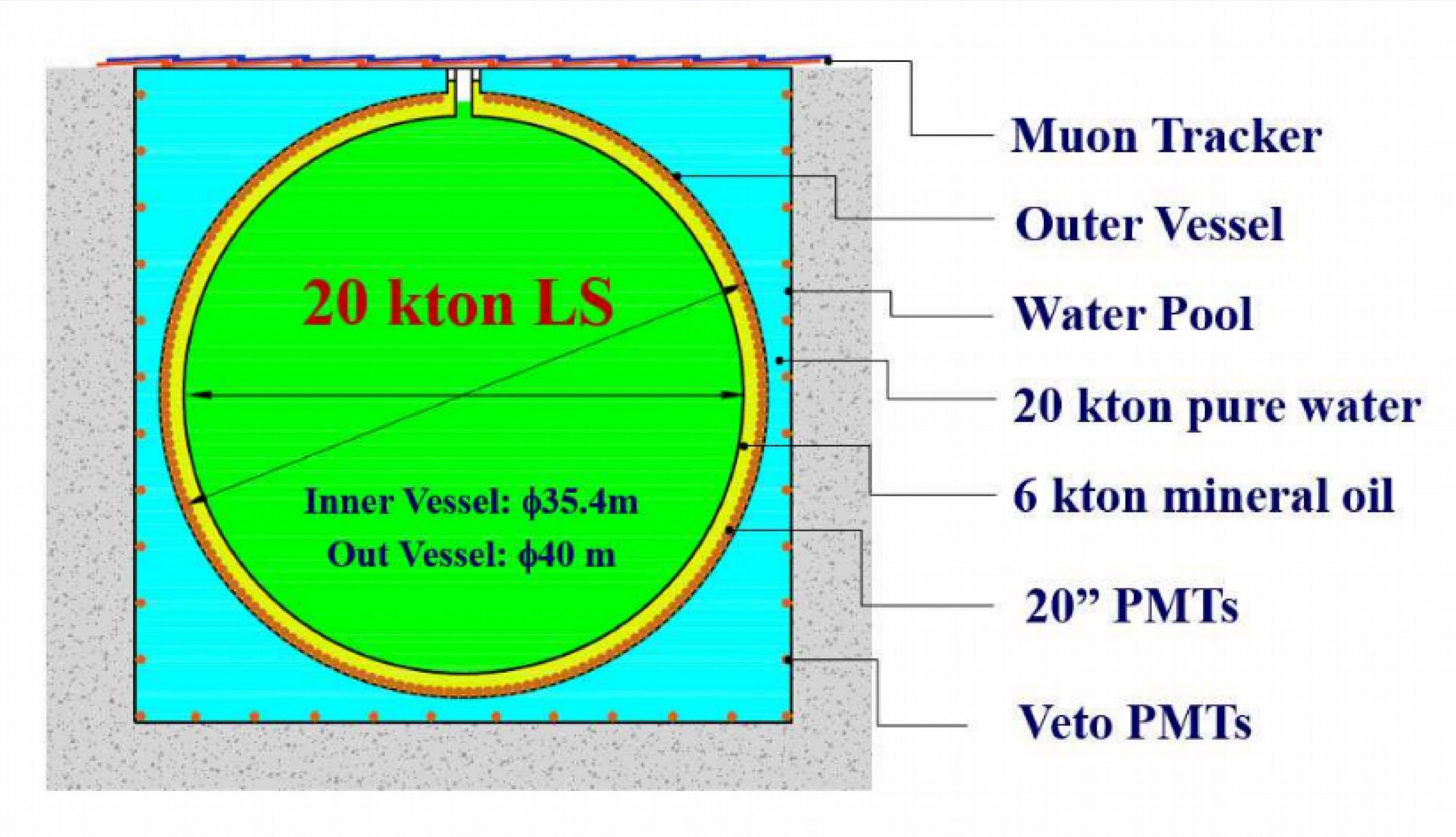
# Extra slides

# JUNO site




[An et al. (2015)]

# JUNO detector




[An et al. (2015)]

# JUNO for $\bar{\nu}_e \to \bar{\nu}_e$

- We simulate data with using the best fit values *with* unitarity
- We fit using the simple χ² fit as follows $\chi^2 \equiv \chi^2_{\rm stat} + \chi^2_{\rm sys}$

8 MeV

Simulated data

$5 \times 35.8 \times 20 = 3.58 \times 10^3$ yr GW kt

$$\chi^2_{\rm stat} \equiv \int_0^{E_{\rm vis}^{\rm max}} dE_{\rm vis} \left( \frac{\dfrac{dN^{\rm obs}}{dE_{\rm vis}} - f_{\rm norm} \displaystyle\sum_{i={\rm reac}} \dfrac{dN_i^{\rm fit}}{dE_{\rm vis}}}{\sqrt{\dfrac{dN^{\rm obs}}{dE_{\rm vis}}}} \right)^2$$

$$\chi^2_{\rm sys} \equiv \left( \frac{1 - f_{\rm norm}}{\sigma_{f_{\rm norm}}} \right)^2$$

3 % and 6 %

$$\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min} = 2.3,\ 6.18 \text{ and } 11.93\ (1,\ 4 \text{ and } 9)$$

$\chi^2_{\rm min} = 0$ by definition

Constraints: $\displaystyle\sum_{i=1}^{3} |U_{ei}|^2 \leq 1,$ $\quad 0 \leq \mathcal{C}_{ee} \leq (1 - \displaystyle\sum_{i=1}^{3} |U_{ei}|^2)^2$

Most conservative case: N →∞

# Understanding the correlations

$$P(\bar{\nu}_e \to \bar{\nu}_e) = \mathcal{C}_{ee} + \left( \sum_j^3 |U_{ej}|^2 \right)^2 - 4 \sum_{k>j}^3 |U_{ej}|^2 |U_{ek}|^2 \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$

Invariant transformations

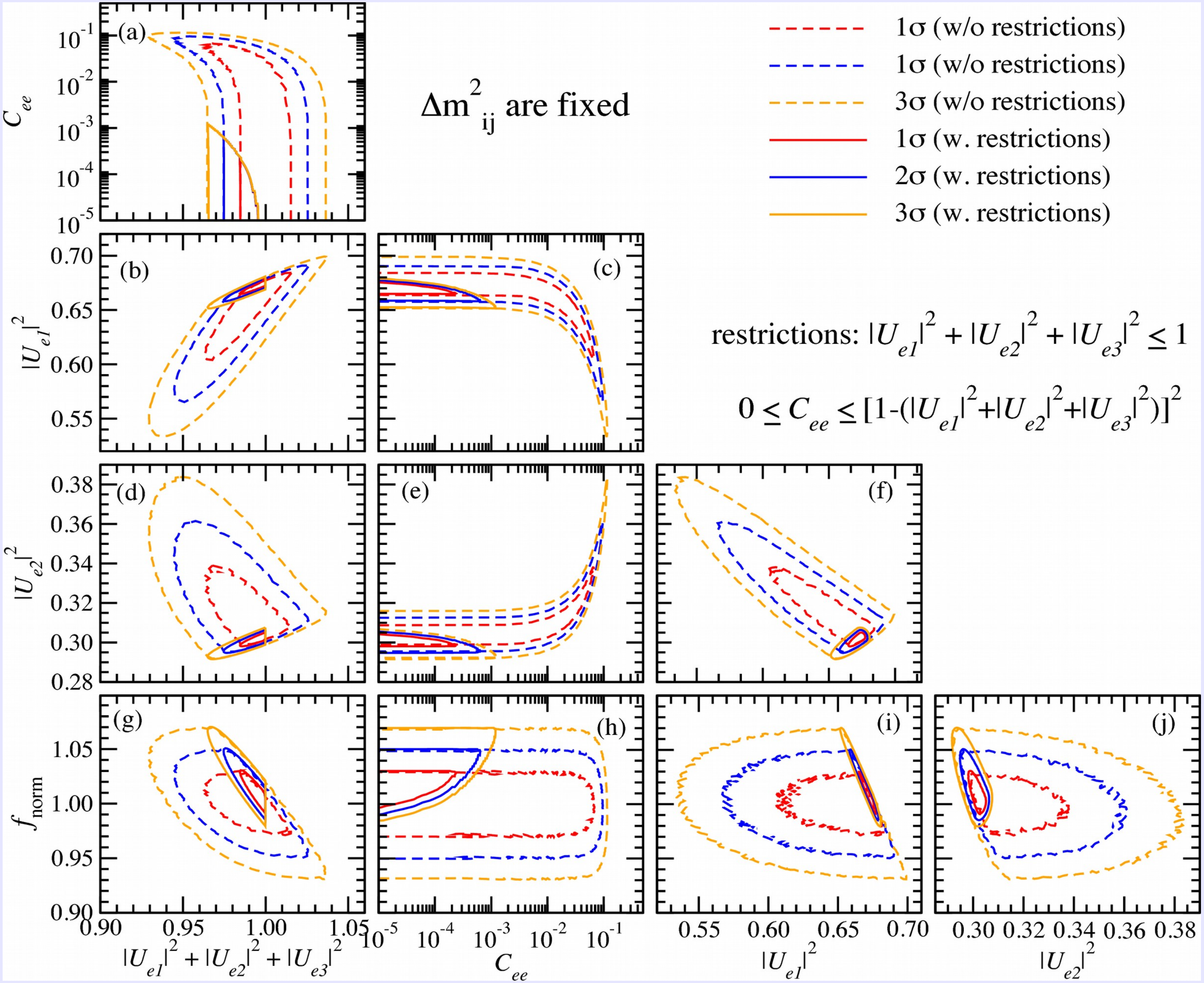
$$\begin{aligned} |U_{ei}|^2 &\to \xi |U_{ei}|^2 \quad (i = 1, 2, 3), \\ f_{\mathrm{norm}} &\to \xi^{-2} f_{\mathrm{norm}}, \\ \mathcal{C}_{ee} &\to \xi^2 \mathcal{C}_{ee} \end{aligned}$$

Broken by

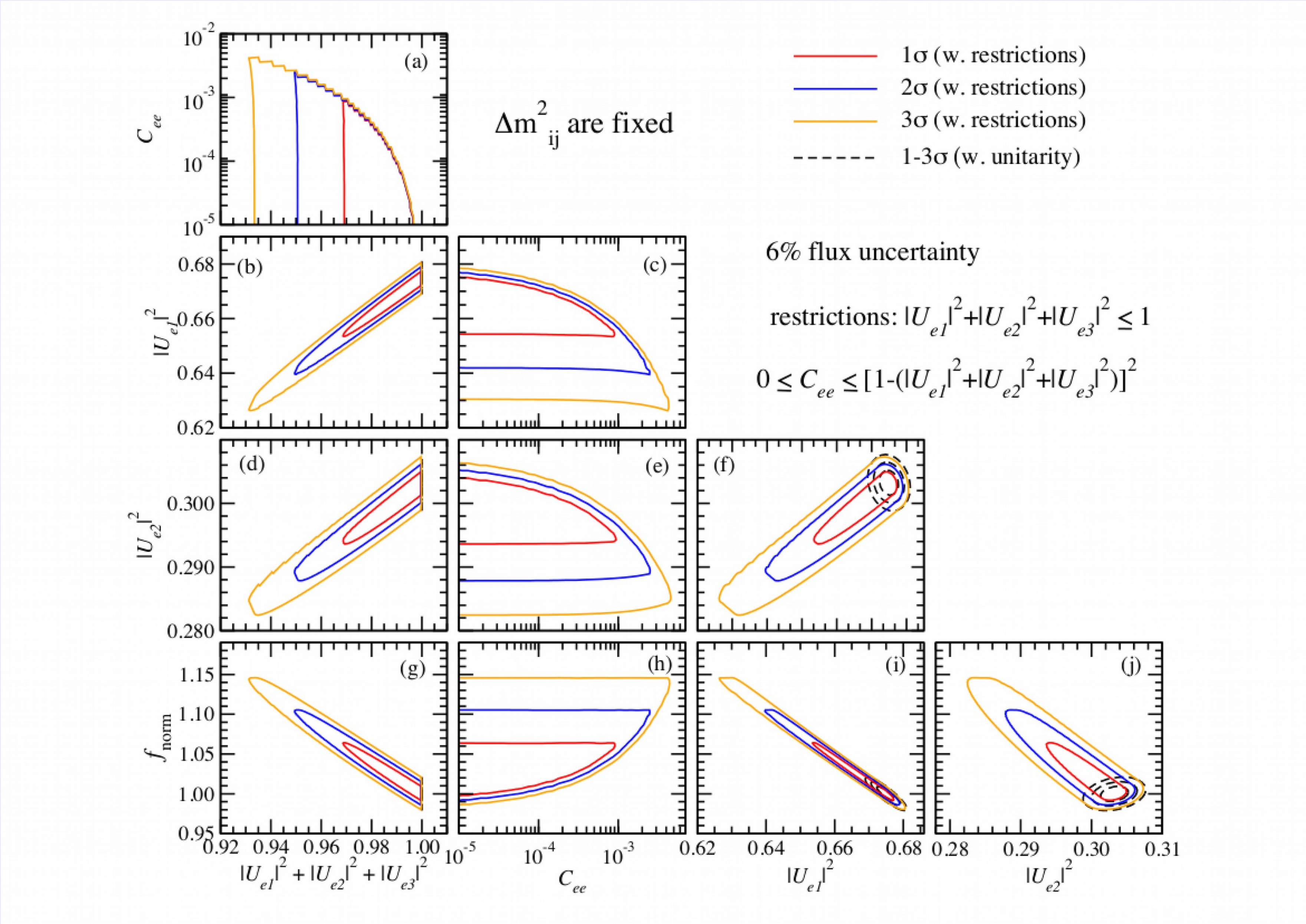
$$\chi^2_{\mathrm{sys}} \equiv \left( \frac{1 - f_{\mathrm{norm}}}{\sigma_{f_{\mathrm{norm}}}} \right)^2$$

$\mathcal{C}_{ee}$ essentially determined by $\mathcal{C}_{ee} \leq (1 - \sum_{i=1}^3 |U_{ei}|^2)^2$

# JUNO (w/o constraints)



# JUNO (6% flux uncertainty)



# CP violation

- 3x3 non-unitarity matrix has $(3\text{-}1)^2=4$ phases
- CP violation can be quantified by

$$\Delta P_{\beta\alpha} \equiv P(\nu_\beta \to \nu_\alpha) - P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) = -4 \sum_{j>i} J_{\alpha\beta ij} \sin\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$$

$$J_{\alpha\beta ij} \equiv \mathrm{Im}\left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\right)$$

- We can rewrite in another form using unitarity relation

$$\sum_i U_{\alpha i} U_{\beta i}^* = \delta_{\alpha\beta} - \sum_I W_{\alpha I} W_{\beta I}^* \Longrightarrow \sum_i J_{\alpha\beta ij} = -\mathrm{Im}\left(U_{\alpha j}^* U_{\beta j} W_{\alpha I} W_{\beta I}^*\right) \equiv S_{\alpha\beta j}$$

$$\begin{aligned}\Delta P_{\beta\alpha} &= -16 J_{\alpha\beta 12} \sin\left(\frac{\Delta m_{32}^2 x}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 x}{4E}\right) \sin\left(\frac{\Delta m_{21}^2 x}{4E}\right) \\ &+ 4 S_{\alpha\beta 1} \sin\left(\frac{\Delta m_{31}^2 x}{2E}\right) + 4 S_{\alpha\beta 2} \sin\left(\frac{\Delta m_{32}^2 x}{2E}\right)\end{aligned}$$

# Matter effects

- We start from Hamiltonian in flavor basis

$$H = \mathbf{U}\,\mathrm{diag}(\Delta_1, \Delta_2, ..., \Delta_{3+N})\,\mathbf{U}^\dagger + \mathrm{diag}(\Delta_A - \Delta_B, -\Delta_B, \Delta_B, 0, ..., 0)$$

$$\Delta_{i(J)} \equiv \frac{m^2_{i(J)}}{2E}, \;\; \Delta_A \equiv \frac{A}{2E}, \;\; \Delta_B \equiv \frac{B}{2E}$$

- with matter potential due to charged and neutral currents

$$A = 2\sqrt{2} G_F N_e E \approx 1.52 \times 10^{-4} \left(\frac{Y_e \rho}{\mathrm{g.cm^{-3}}}\right) \left(\frac{E}{\mathrm{GeV}}\right) \mathrm{eV}^2$$

$$B = \sqrt{2} G_F N_n E = \frac{1}{2} \left(\frac{N_n}{N_e}\right) A$$

- We do not make assumption on size of W but matter effects as perturbation

$$|A| \ll |\Delta m^2_{31}|$$

# Matter effects (as perturbations)

- We rotate to mass basis

$$\tilde{H} \equiv \mathbf{U}^\dagger H \mathbf{U} = \tilde{H}_0 + \tilde{H}_1$$

diagonal

$$S(x) = \mathbf{U}\tilde{S}(x)\mathbf{U}^\dagger$$

$$\tilde{S}(x) = T\exp\left[-i\int_0^x dx'\tilde{H}(x')\right]$$

- Define

$$\tilde{S}(x) \equiv e^{-i\tilde{H}_0 x}\Omega(x) \implies i\frac{d}{dx}\Omega(x) = H_1(x)\Omega(x), \qquad H_1(x) \equiv e^{i\tilde{H}_0 x}\tilde{H}_1 e^{-i\tilde{H}_0 x}$$

- We can solve perturbatively

$$\Omega(x) = 1 + (-i)\int_0^x dx' H_1(x') + (-i)^2\int_0^x dx' H_1(x')\int_0^{x'} dx'' H_1(x'') + \cdots$$

- Oscillation probability

$$P(\nu_\beta \to \nu_\alpha) = |S_{\alpha\beta}|^2 = \left|S_{\alpha\beta}^{(0)} + S_{\alpha\beta}^{(1)}\right|^2 = \left|S_{\alpha\beta}^{(0)}\right|^2 + 2\,\mathrm{Re}\left[S_{\alpha\beta}^{(0)*}S_{\alpha\beta}^{(1)}\right]$$

vacuum term

# Matter effects (as perturbations)

- We have terms like

$$2\sum_j \sum_l \sum_K |U_{\alpha j}|^2 \mathrm{Re}\left[\Delta_A W_{\alpha K} U^*_{\alpha l} W^*_{eK} U_{el} - \Delta_B W_{\alpha K} U^*_{\alpha l} \sum_\gamma W^*_{\gamma K} U_{\gamma l}\right] \frac{\cos(\Delta_l - \Delta_j)x}{(\Delta_l - \Delta_K)}$$

- To make the model insensitive to sterile mass spectra, besides partial decoherence $0.1\ \mathrm{eV}^2 \lesssim m_J^2 \lesssim 1\ \mathrm{MeV}^2$, we require also

$$\frac{|\Delta_A|}{(\Delta_J - \Delta_k)} = \frac{|A|}{\Delta m^2_{Jk}} \ll |W|^2$$

$$\frac{|A|}{\Delta m^2_{Jk}} = 2.13 \times 10^{-3} \left(\frac{\Delta m^2_{Jk}}{0.1\mathrm{eV}^2}\right)^{-1} \left(\frac{\rho}{2.8\,\mathrm{g/cm}^3}\right) \left(\frac{E}{1\ \mathrm{GeV}}\right)$$

- This is fulfilled unless W is very small

$$W^2 \sim 10^{-3} (W^2 \lesssim 10^{-n}) \implies \Delta m^2_{Jk} \simeq m_J^2 \gtrsim 1\,\mathrm{eV}^2 (10^{(n-3)}\,\mathrm{eV}^2)$$

# Matter effects (as perturbations)

- Finally for disappearance channels, corrections from matter effects give

$$P(\nu_\alpha \to \nu_\alpha)^{(1)} = -2\sum_{j\neq k}|U_{\alpha j}|^2|U_{\alpha k}|^2 \sin(\Delta_k - \Delta_j)x \left[(\Delta_A x)|U_{ek}|^2 - (\Delta_B x)\sum_\gamma |U_{\gamma k}|^2\right]$$

$$+4\sum_j\sum_{k\neq l}|U_{\alpha j}|^2 \mathrm{Re}\left[\Delta_A U_{\alpha k}U^*_{\alpha l}U^*_{ek}U_{el} - \Delta_B U_{\alpha k}U^*_{\alpha l}\sum_\gamma U^*_{\gamma k}U_{\gamma l}\right]$$

$$\times \frac{\sin^2\frac{(\Delta_k-\Delta_j)x}{2} - \sin^2\frac{(\Delta_l-\Delta_j)x}{2}}{(\Delta_l - \Delta_k)}$$

- Similarly for appearance (but more lengthy expressions)
- The important effect of UV comes only in the vacuum term
- Can the 3+N model be insensitive to details of the sterile sector under strong matter effects? (work in progress)

# Matter effects (as perturbations)

- For appearance channels, corrections from matter effects give

$$P(\nu_\beta \to \nu_\alpha)^{(1)}$$

$$= 2\sum_{j\neq k}\left[-\mathrm{Re}\left(U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta k}\right)\sin(\Delta_k-\Delta_j)x + \mathrm{Im}\left(U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta k}\right)\cos(\Delta_k-\Delta_j)x\right]$$

$$\times\left[(\Delta_A x)|U_{ek}|^2 - (\Delta_B x)\sum_\gamma |U_{\gamma k}|^2\right]$$

$$+2\sum_j\sum_{k\neq l}\mathrm{Re}\left[\Delta_A U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta l}U^*_{ek}U_{el} - \Delta_B U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta l}\sum_\gamma U^*_{\gamma k}U_{\gamma l}\right]$$

$$\times\frac{\cos(\Delta_l-\Delta_j)x - \cos(\Delta_k-\Delta_j)x}{(\Delta_l-\Delta_k)}$$

$$+2\sum_j\sum_{k\neq l}\mathrm{Im}\left[\Delta_A U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta l}U^*_{ek}U_{el} - \Delta_B U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta l}\sum_\gamma U^*_{\gamma k}U_{\gamma l}\right]$$

$$\times\frac{\sin(\Delta_l-\Delta_j)x - \sin(\Delta_k-\Delta_j)x}{(\Delta_l-\Delta_k)}$$