Wave-Packet Treatment for Detection of Accelerator Neutrinos

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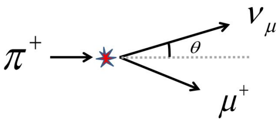
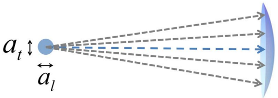
Introduction

- Quantum particles should be treated as **Wave Packets** (WPs).
  - Finite size of the WP introduces intrinsic momentum uncertainty.
  - Non-zero probability of detecting the neutrino off its classical path.
  - $\Delta\theta \sim \Delta p_\perp / p_0 \sim 1/2E_\nu a_t$
- Current Monte Carlo simulations assume **Point-like Particles** (PPs)
  - In particular, "classical" pion decay in lab frame is used:

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} \approx \frac{\gamma^2 \left(1+\tan^2\theta\right)^{\frac{3}{2}}}{\pi\left(1+\gamma^2\tan^2\theta\right)^2} \ \& \ E_\nu(\theta) \approx \frac{\left(1-m_\mu^2/M_\pi^2\right)\gamma M_\pi}{1+\gamma^2\tan^2\theta}$$

  - The above describe the **MEAN PATH** and energy of a neutrino WP.





Motivation & Approach

Motivation:

- Focus v.s. Defocus of the neutrino beam. Does WP treatment change the prediction of experimental observables?
- Is it possible to determine the WP size from accelerator experiments?

Approach:

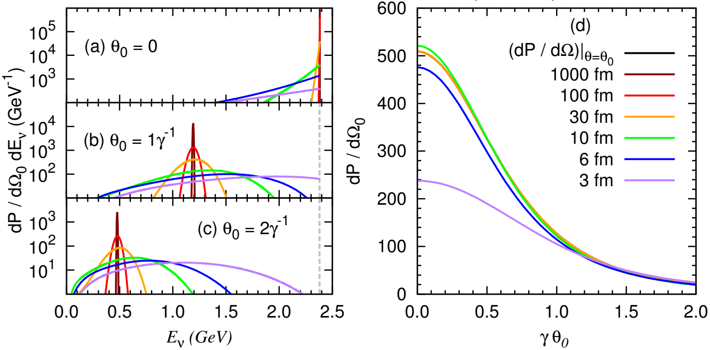
- We assume **3D** and **Massless** Gaussian WP parameterized by a_l and a_t. Its momentum distribution is assumed to be sharp.
- We derive the probability $\Theta(\theta')$ of detecting the neutrino at an angle θ' relative to its classical path. $\Theta(\theta') \sim \exp\{-\frac{\theta'^2}{2\cdot(2E_\nu a_t)^{-2}}\}$.
- With $\Theta(\theta')$, we derive the probability distribution as a function of neutrino energy and observation angle relative to the pion's mean trajectory.
- The new distribution is applied to calculate experimental spectrum.

Modified Probability Distribution

- Due to WP spreading, the probability of detecting the neutrino within $\mathrm{d}\Omega_0$ is an incoherent sum over different emission directions.

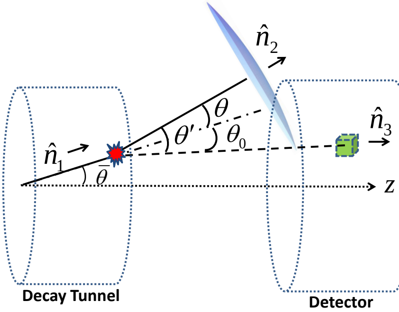
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega_0 \mathrm{d}E_\nu} = \int_0^{2\pi} \mathrm{d}\phi \, \frac{\mathrm{d}(\cos\theta)}{\mathrm{d}E_\nu} \frac{\mathrm{d}P}{\mathrm{d}\Omega} \Theta(\theta', E_\nu)$$

- WP and PP treatments are equivalent if $(2E_\nu a_t)^{-1} \ll \gamma^{-1}$.



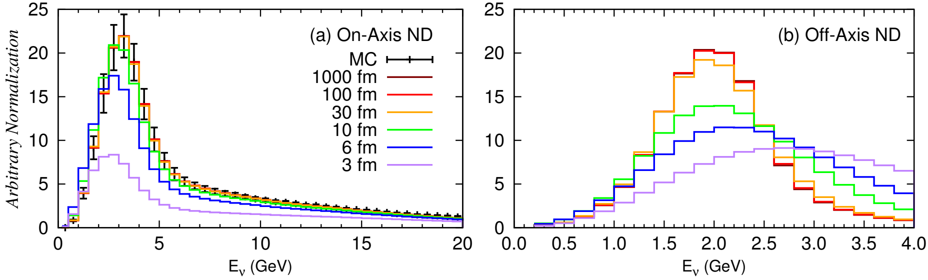
Application to Accelerator Experiments

- For demonstration purpose, we consider secondary beam (π^+ only) profiles and near detector geometries **similar** to the MINOS and NOνA experiments.
- Geometric variables in the numerical calculation. The azimuthal angles are not displayed.



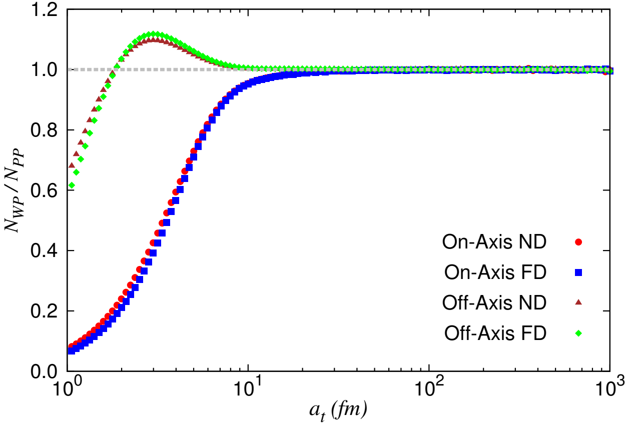
Application to Accelerator Experiments

- Predicted ν_μ charged current spectrum in the near detectors of (a) on-axis and (b) off-axis experiments.
  - MC simulation with uncertainty from PRL 106, 181801 (2011) is included in (a) for comparison. Caution: no statistical interpretation is intended here!
  - WP spreading shifts the spectrum toward low (high) energy in the on-axis (off-axis) experiment.



Application to Accelerator Experiments

- N_{WP}/N_{PP} as a function of a_t.
  - The number is counted regardless of neutrino energy.
  - Assume no neutrino oscillations in the far detector calculation.
  - Almost the same ratio in both near and far detectors.



Summary

- With a simple Gaussian neutrino WP emerging from pion decay in flight, we derive the modified probability distribution which can be easily included in Monte Carlo simulations.
- WP spreading shifts neutrino spectrum in the opposite directions for on/off-axis experiments.
- Null observation of the spectral shift in the near detector could place a lower bound on a_t.

Thank You!

Massless 3D Gaussian WP

- At $t = 0$, the initial WP can be expressed as

$$\Psi(\vec{r},0) = \frac{1}{(2\pi)^{3/4}\, a_t a_l^{1/2}} \exp\left(-\frac{\rho^2}{4a_t^2} - \frac{z^2}{4a_l^2} + ip_0 z\right).$$

- The probability $\Theta(\theta')$ can be found by two equivalent methods:
  1. By solving the wave equation, $\Psi(\vec{r}, t > 0)$ turns out to be a spherical wave front with constant radial width a_l and an asymptotically constant angular distribution. The wave front moves at the speed of light.
  2. Alternatively, one can analyze the momentum distribution $\tilde{\Psi}(\vec{p})$ of the initial WP. The normalization condition of $\tilde{\Psi}(\vec{p})$, $\Theta(\theta')$ suggests.

$$1 = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}|\tilde{\Psi}(\vec{p})|^2 = \int \mathrm{d}\Omega' \underbrace{\int_0^\infty \frac{p^2\mathrm{d}p}{(2\pi)^3}|\tilde{\Psi}(\vec{p})|^2}_{\Theta(\theta')} \tag{1}$$

Spectral Shift & Adjusted Beam Normalization

- Define detector position according to the characteristic angle of pion decay.
- Assume collinear trajectories for all pions for simplicity.
  - At inside (outside) position, the detector sees less (more) number of neutrinos. The measured neutrinos are less (more) energetic.
  - An on-axis detector is always at the inside position.
  - An off-axis detector can be either "inside" or "outside", depending on the pion energy.