SIBYLL 2.3 and MCEq, a versatile numerical solver for atmospheric lepton fluxes

Anatoli Fedynitch[1,2] ,Ralph Engel[1],Thomas K. Gaisser[3], Felix Riehn[1] and Todor Stanev[3]

(1) *KIT (IKP)*

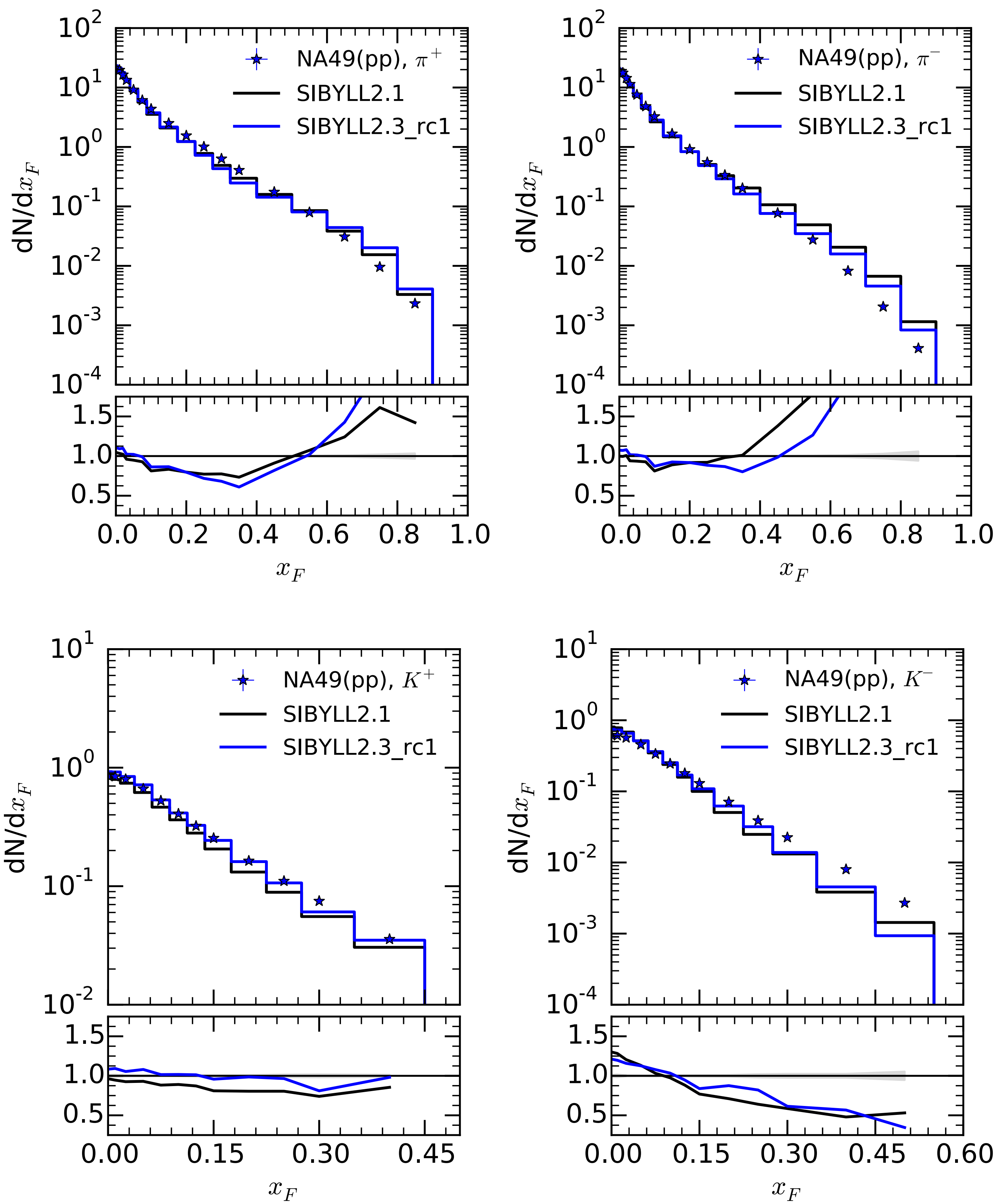
(2) CERN

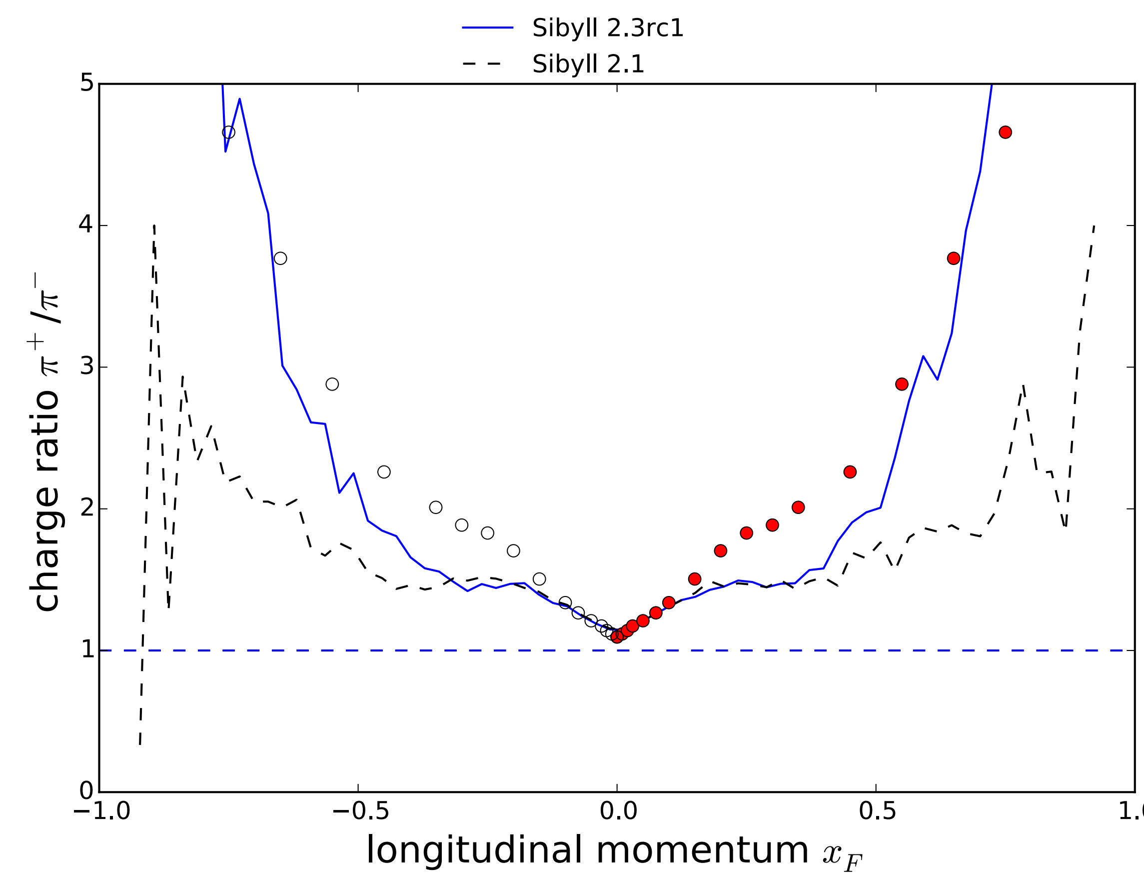
(3) Bartol Research Institute, University of Delaware, Newark, USA

What's new in SIBYLL 2.3?

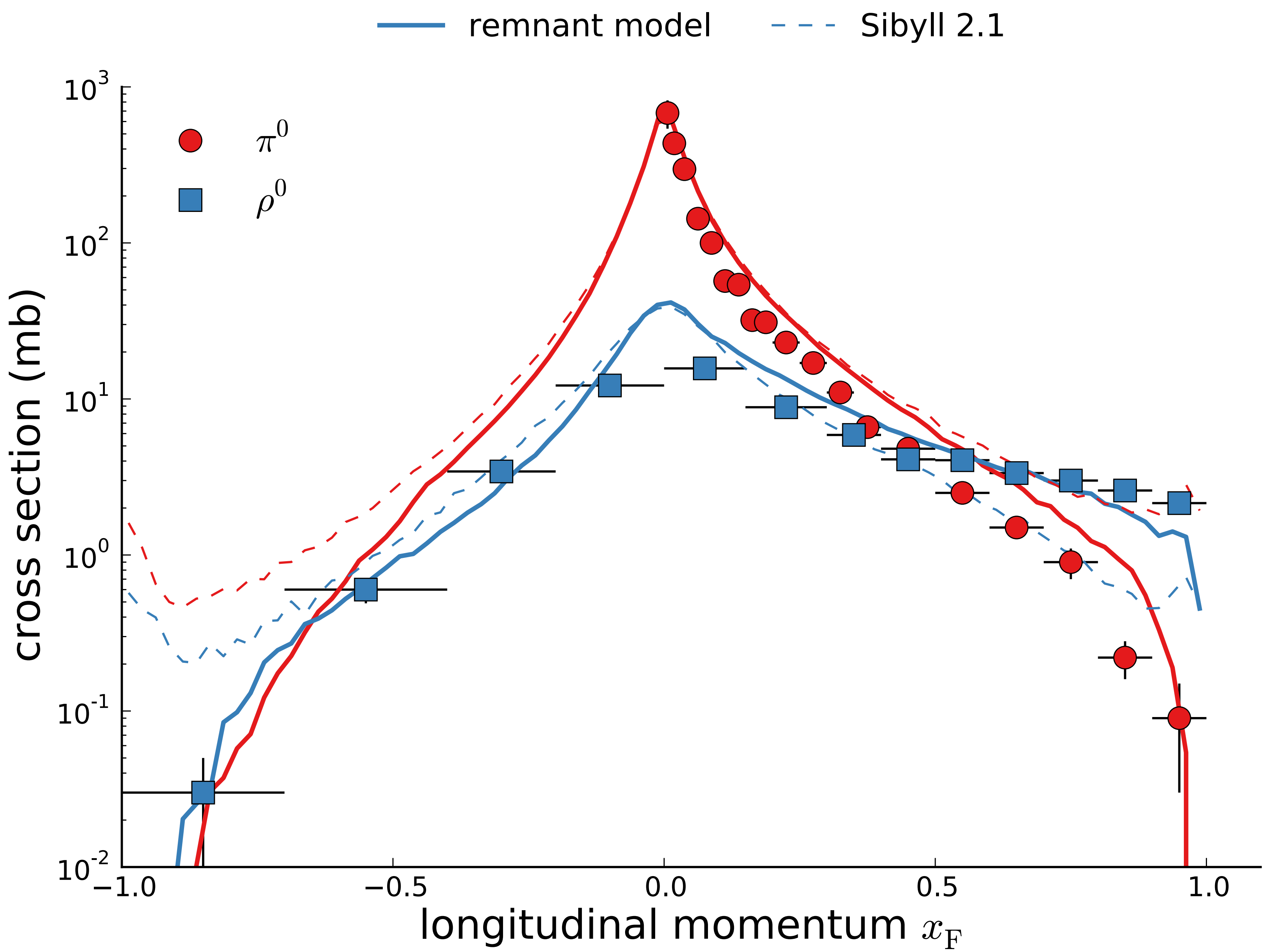
- development by Felix Riehn and Ralph Engel (KIT)
- baryon fragmentation
- generation of charmed hadrons
- improved vector meson production
- careful tuning to fixed-target and collider data
- at the same time verification using atmospheric muon data with **MCEq**
- many bugs fixed
- publicly available this summer

Pions and Kaons





Vector mesons



Charm production mechanisms

Contribution from hard scattering

- Largest contribution at high energies
- many NLO calculations available

Non–perturbative component

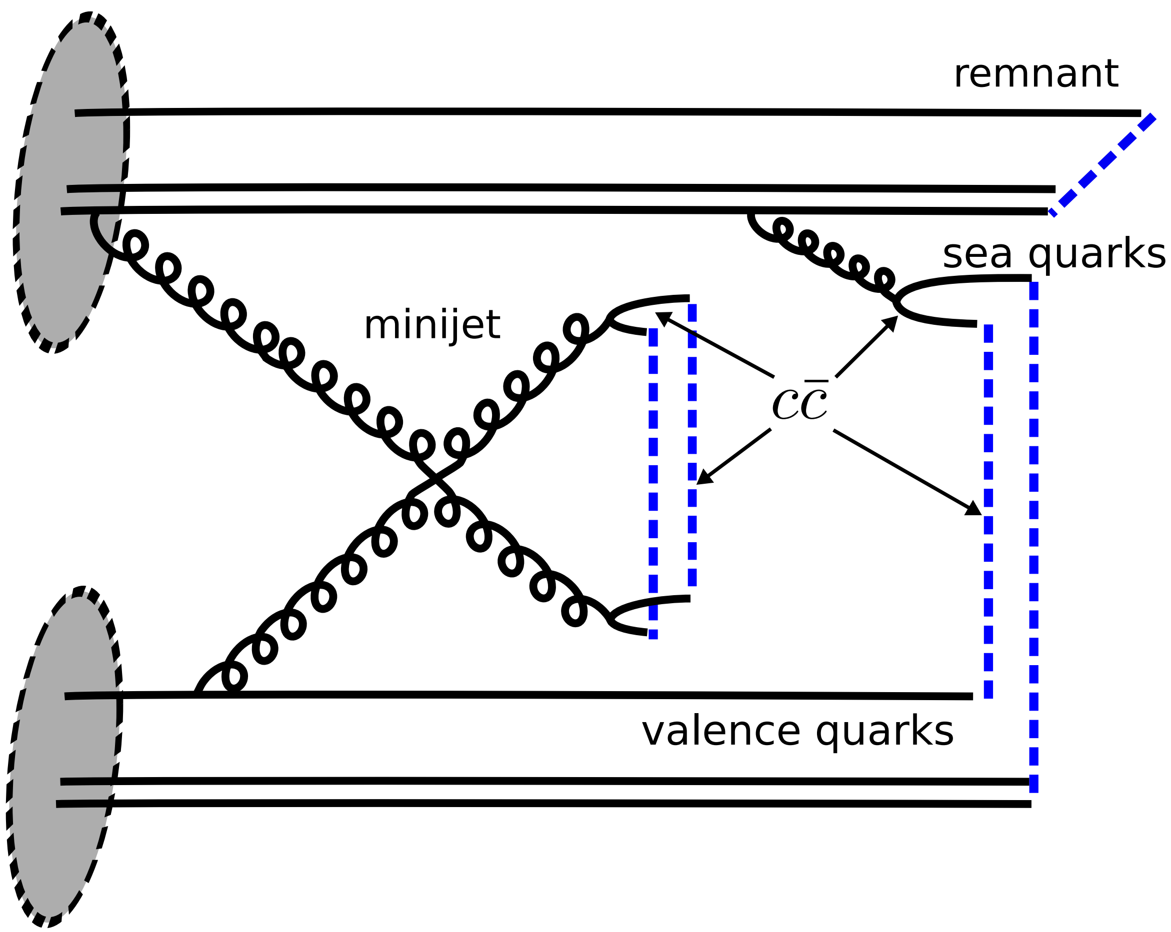
- di-quark fragments together with charm quark in valence scattering
- leading particle effect (SELEX)
- relevant contribution for inclusive fluxes of muons and neutrinos

Charm in fragmenation

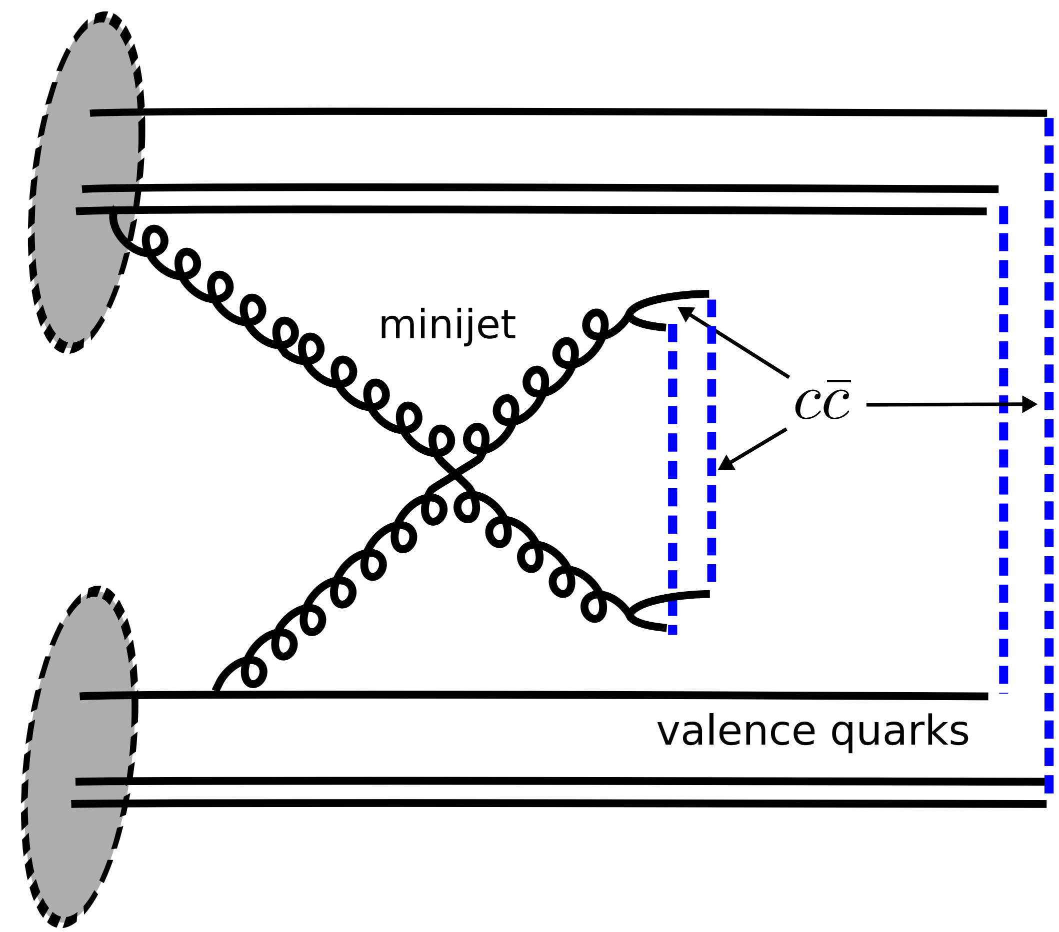
- usually strongly suppressed u:d:s:c = 1:1:0.3:10^{-11}
- in DPMJET-II.55 enhanced by adding higher probability raising charm from the sea close to string ends

Charm production mechanisms

With remnant



No remnant



Figures: F. Riehn (PhD thesis)

LHCb phase-space, how limiting is limited?

- 7TeV c.m energy well beyond the knee

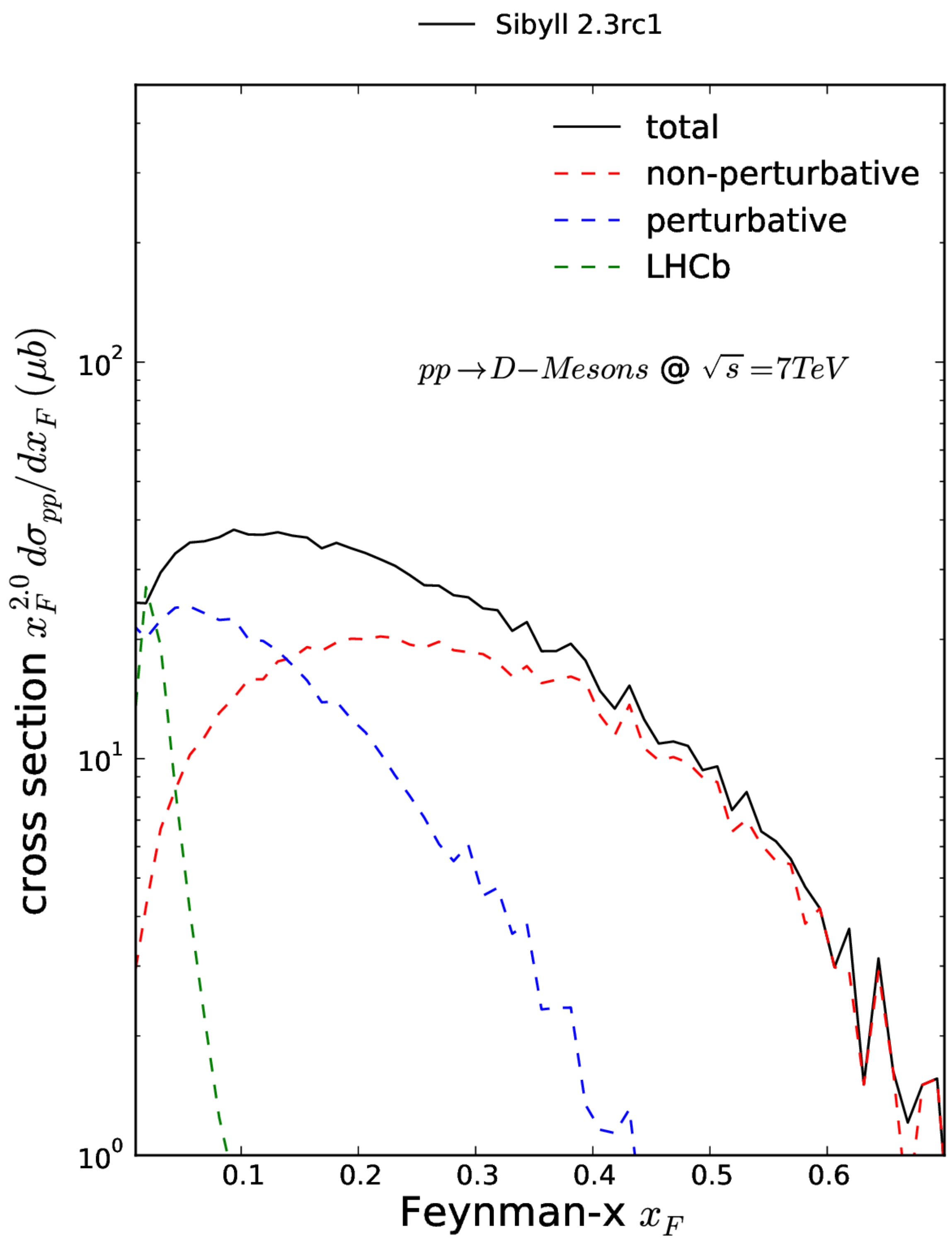
$$\sqrt{s} = 7TeV \rightarrow E_{lab} = 26PeV$$

$$\gamma_{CR} \approx 3$$

- How much does LHCb phasespace contribute to integrated spectrum?

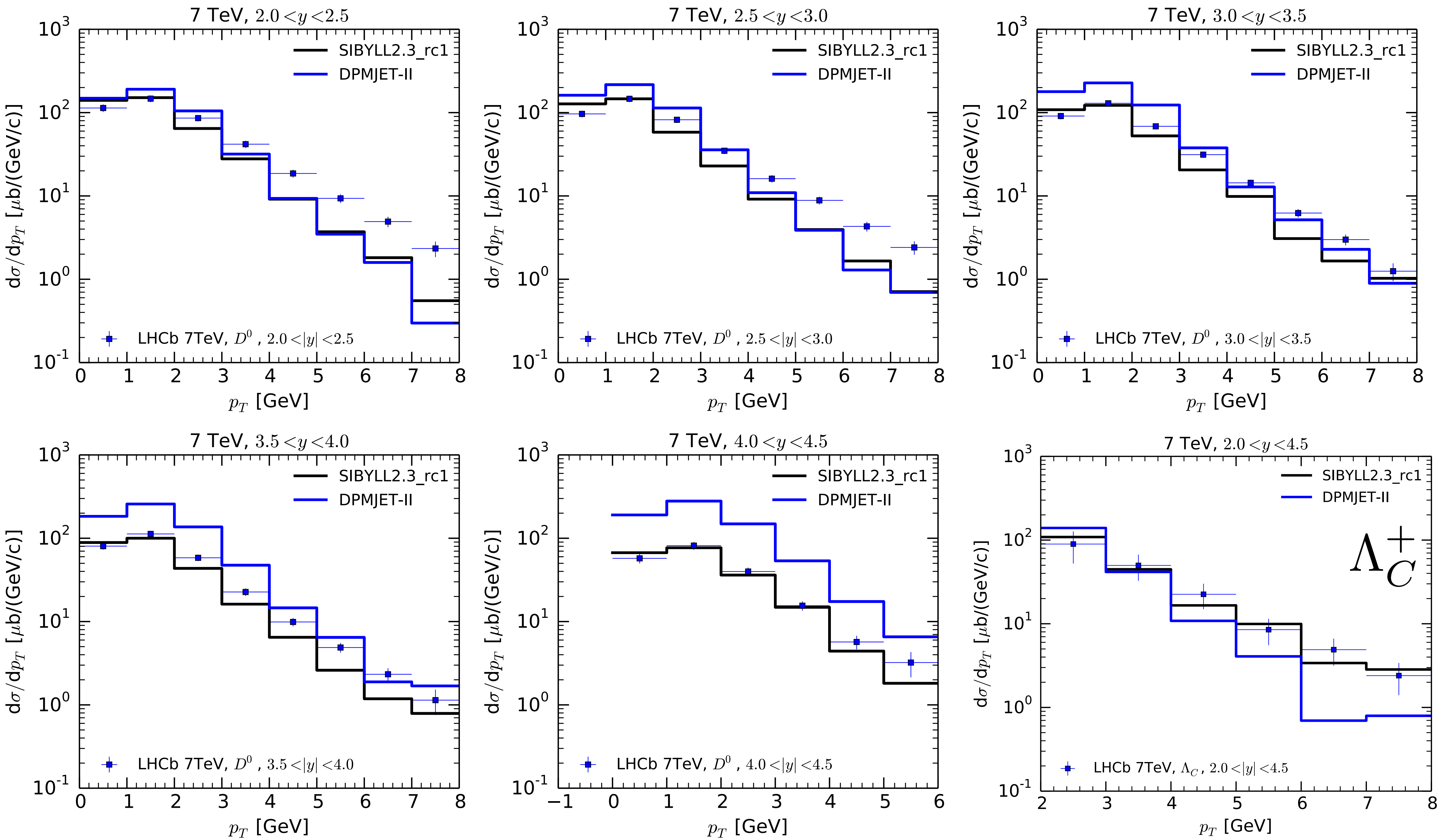
	%
LHCb	7
perturbative	37
Non-perturbative	59

→ LHC data **not** restrictive



F. Riehn, R. Engel, AF, T. Gaisser, T. Stanev, ISVHECRI 2014

LHCb D^0-mesons and Lambda-c

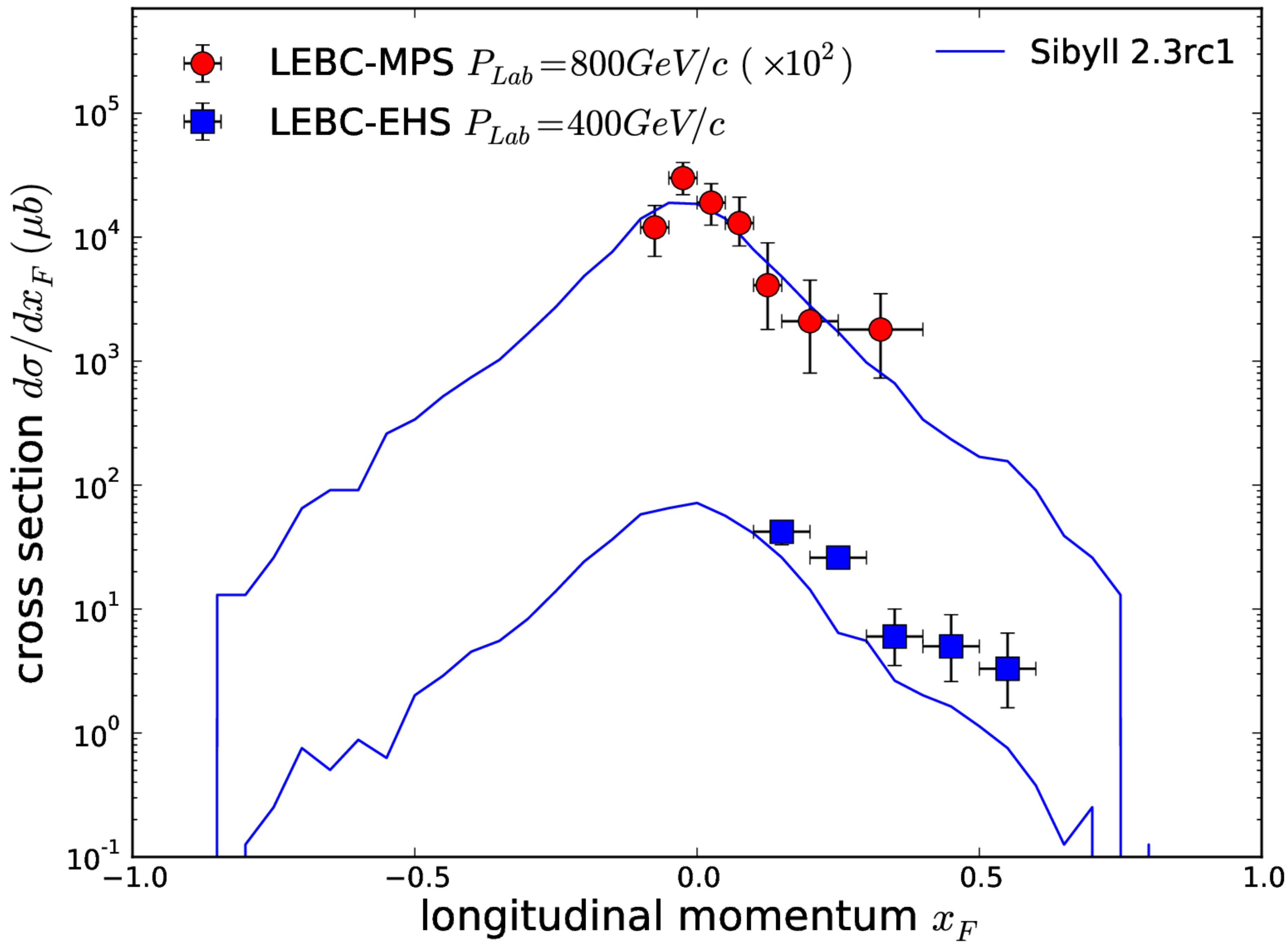


x_F distributions at fixed target experiments

$$\sqrt{s} = 27/39GeV$$

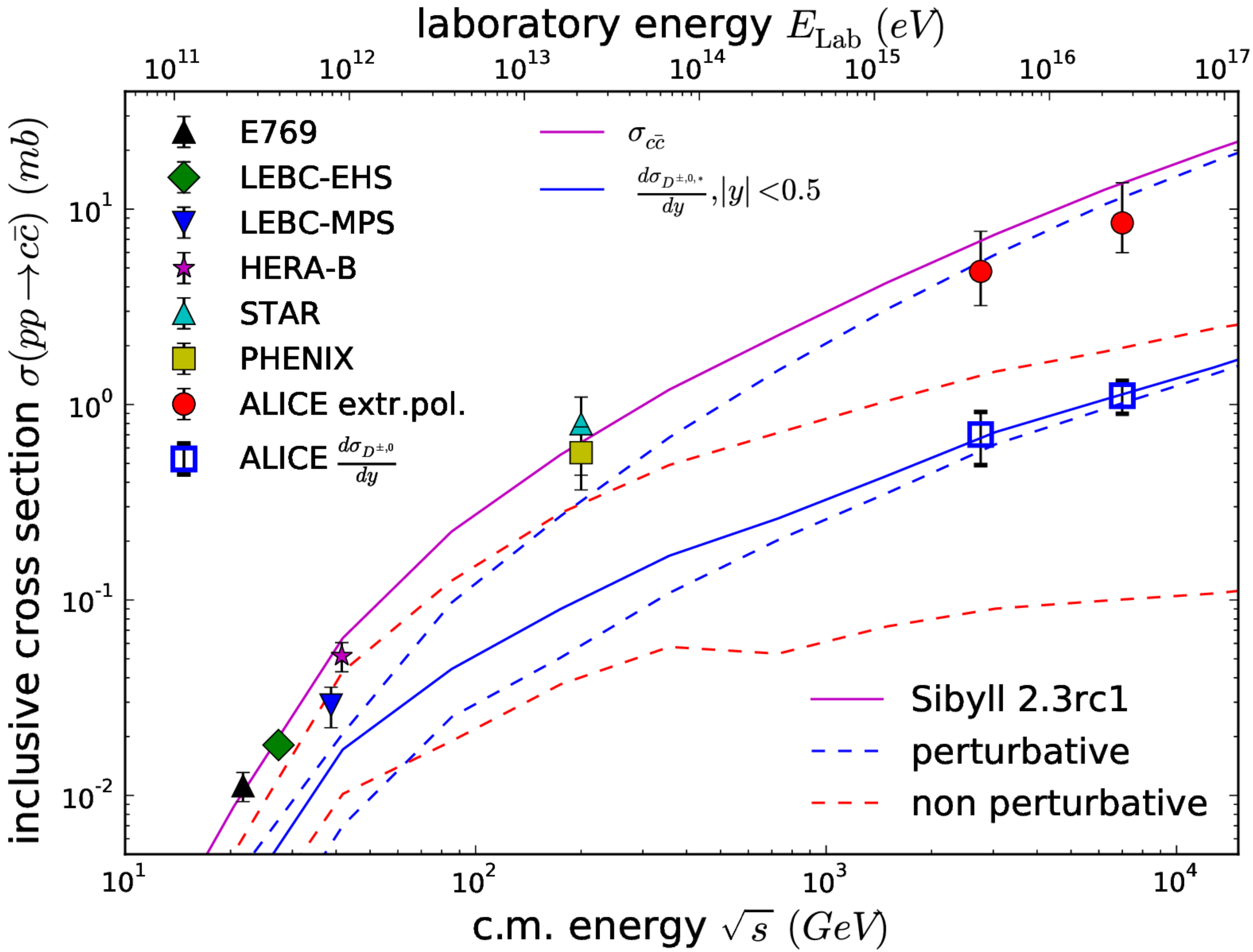
Energy low but full phasespace coverage possible!

$$y_{max}^{39GeV} = 3.7$$



F. Riehn, R. Engel, AF, T. Gaisser, T. Stanev, ISVHECRI 2014

Inclusive charm production



F. Riehn, R. Engel, AF, T. Gaisser, T. Stanev, ISVHECRI 2014

Matrix Cascade Equation - MCEq

- numerical solver for full system of coupled hadronic cascade equations
- very high execution speed due to sparse linear algebra
- sophisticated treatment of short-lived particles (prompt component)
- all physics parameters and inputs for flux calculations are transparent to users
- public open-source code

Ideal tool for background and systematics calculations of atmospheric lepton fluxes

Particles in SIBYLL

Leptons

$\mu^+,\ \mu^-,\ \tau^+,\ \tau^-,\ \nu_e,\ \nu_\mu,\ \nu_\tau,\ \bar{\nu}_e,\ \bar{\nu}_\mu,\ \bar{\nu}_\tau$

Mesons

$$K^+,\ K^-,\ K_L^0,\ K_S^0,\ \pi^+,\ \pi^-,\ D^+,\ D^-,\ D^0,\ \bar{D}^0,\ D_s^+,\ D_s^-,\ K^{*+},\ K^{*-},$$
$$K^{*0},\ \bar{K}^{*0},\ D^{*+},\ D^{*-},\ D^{*0},\ \bar{D}^{*0},\ \eta,\ \eta^*,\ \eta_C,\ J/\Psi,\ \omega,\ \phi,\ \pi^0,\ \rho^+,\ \rho^-,\ \rho^0$$

Baryons

$$p,\ \bar{p},\ n,\ \bar{n},\ \Delta^+,\ \Delta^{++},\ \bar{\Delta}^{++},\ \bar{\Delta}^+,\ \Delta-,\ \bar{\Delta}^+,\ \Delta^0,\ \bar{\Delta}^0,\ \Lambda^0,\ \bar{\Lambda}^0,\ \Omega^-,\ \bar{\Omega}^+,$$
$$\Sigma^{*+},\ \bar{\Sigma}^{*-},\ \Sigma^{*-},\ \bar{\Sigma}^{*+},\ \Sigma^{*0},\ \bar{\Sigma}^{*0},\ \Sigma^+,\ \bar{\Sigma}^-,\ \Sigma^0,\ \bar{\Sigma}^0,\ \Lambda_C^+,\ \bar{\Lambda}_C^-,\ \Omega_C^0,\ \bar{\Omega}_C^0,\ \Sigma^-,$$
$$\bar{\Sigma}^+,\ \Xi^-,\ \bar{\Xi}^+,\ \Xi^0,\ \bar{\Xi}^0,\ \Xi_C^+,\ \bar{\Xi}_C^+,\ \Xi_C^0,\ \bar{\Xi}_C^0,\ \Sigma_C^{*+},\ \Sigma_C^{*++},\ \bar{\Sigma}_C^{*--},\ \bar{\Sigma}_C^{*-},\ \Sigma_C^{*0},$$
$$\bar{\Sigma}_C^{*0},\ \Sigma_C^+,\ \Sigma_C^{++},\ \bar{\Sigma}_C^{--},\ \bar{\Sigma}_C^-,\ \Sigma_C^0,\ \bar{\Sigma}_C^0,\ \Xi^{*-},\ \bar{\Xi}^{*+},\ \Xi^{*0},\ \bar{\Xi}^{*0}$$

Very different particles can contribute and become important at high energies

Matrix form I

(discretized) coupled cascade equation

For more details, see arXiv:1503.00544

for hadron of type h at (grid-) energy Ei:

$$\frac{\mathrm{d}\phi_h(E_i)}{\mathrm{d}X} = -\frac{\phi_h(E_i)}{\lambda_{int}^{(h)}(E_i)} + \sum_{E_k \geq E_i} \sum_k \frac{c_{k\to h}(E_i, E_k)}{\lambda_{int}^{(k)}(E_k)} \phi_k(E_k) - \frac{\phi_h(E_i)}{\lambda_{dec}^{(h)}(E_i, X)} + \sum_{E_k \geq E_i} \sum_k \frac{d_{k\to h}(E_i, E_k)}{\lambda_{dec}^{(k)}(E_k, X)} \phi_k(E_k)$$

flux state vector

$$\vec{\phi} = \begin{pmatrix} \phi_p(E_0) \\ \phi_p(E_1) \\ \cdots \\ \phi_p(E_N) \\ \phi_n(E_0) \\ \cdots \\ \phi_n(E_N) \\ \phi_\pi^+(E_0) \\ \cdots \\ \phi_{\bar{\nu}_e}(E_N) \end{pmatrix}$$

$E_0 ... E_N$ — energy grid, typical 7-8 bins/decade

$c_{k\to h}(E_i, E_k)$ — inclusive hadron production cross-section from hadronic interaction models

$d_{k\to h}(E_i, E_k)$ — inclusive decay cross-section from PYTHIA 8 Monte Carlo

Matrix form II

2 dimensional representation of coupled CE

$$(\text{hadrons} \times \text{E} - \text{grid}) \times (\text{hadrons} \times \text{E} - \text{grid})$$

inclusive production cross-sections of h in interactions of l (sampling of interaction models)

$$\boldsymbol{C}_{l\to h} = \begin{pmatrix} c_{l(E_0)\to h(E_0)} & \cdots & c_{l(E_0)\to h(E_N)} \\ & & c_{l(E_1)\to h(E_N)} \\ & \ddots & \vdots \\ 0 & & c_{l(E_N)\to h(E_N)} \end{pmatrix}$$

interaction rates

$$\boldsymbol{\Lambda_{int}} = \text{diag}(\frac{1}{\lambda^p_{int,E_0}} \cdots \frac{1}{\lambda^p_{int,E_N}}, \frac{1}{\lambda^n_{int,E_0}}, \cdots, \frac{1}{\lambda^n_{int,E_N}}, \frac{1}{\lambda^{\pi^+}_{int,E_0}}, \cdots),$$

interaction matrix

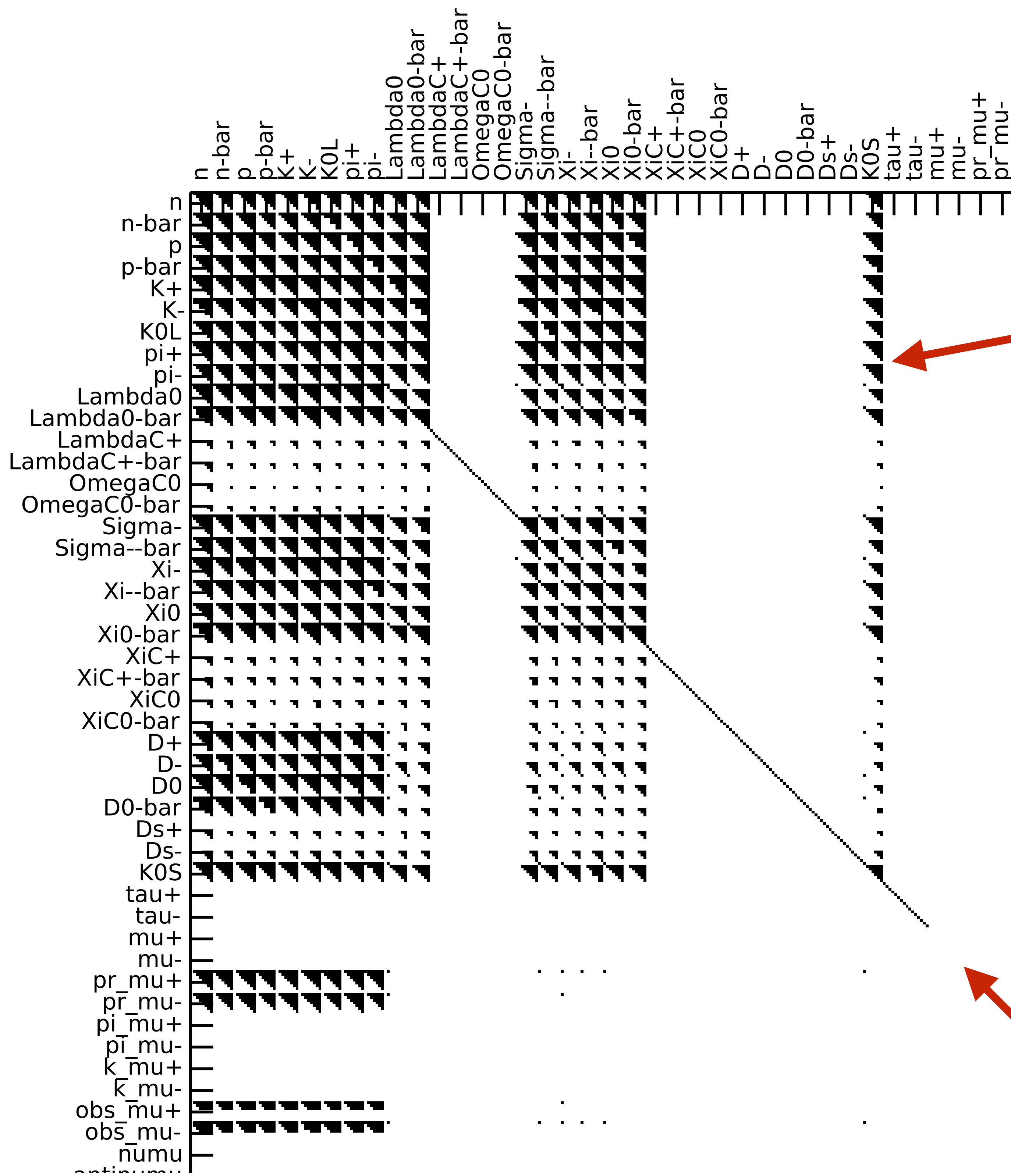
$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{C}_{p\to p} & \boldsymbol{C}_{n\to p} & \boldsymbol{C}_{\pi^+\to p} & \cdots \\ \boldsymbol{C}_{p\to n} & \boldsymbol{C}_{n\to n} & \boldsymbol{C}_{\pi^+\to n} & \cdots \\ \boldsymbol{C}_{p\to \pi^+} & \boldsymbol{C}_{n\to \pi^+} & \boldsymbol{C}_{\pi^+\to \pi^+} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Same approach for decays

Matrix-form

$$\frac{\mathrm{d}}{\mathrm{d}X}\boldsymbol{\phi} = \left[(-\boldsymbol{1} + \boldsymbol{C})\boldsymbol{\Lambda}_{int} + \frac{1}{\rho(X)}(-\boldsymbol{1} + \boldsymbol{D})\boldsymbol{\Lambda}_{dec}\right]\boldsymbol{\phi}.$$

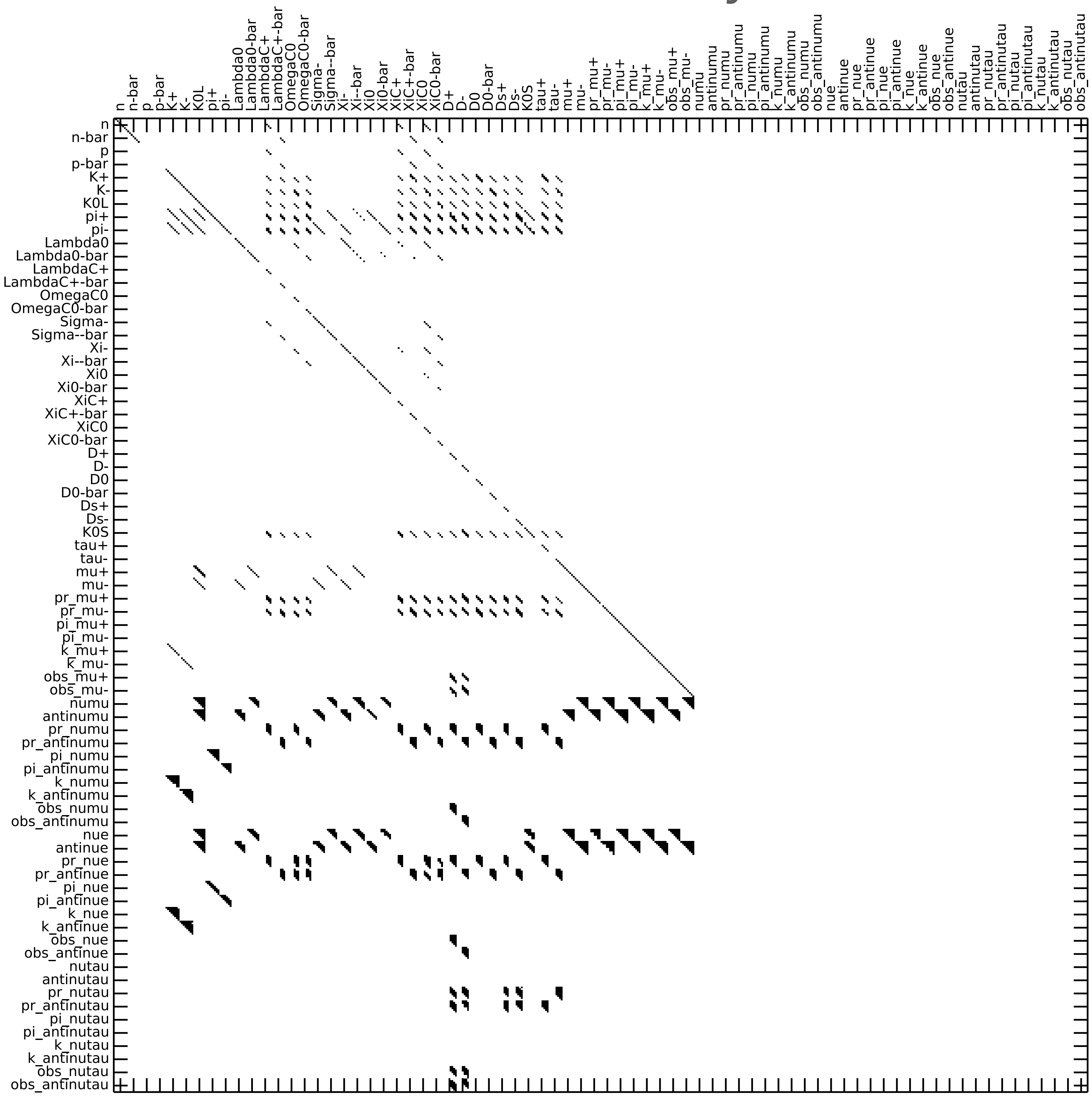
Interaction matrix



$$\frac{c_{k\to h}(E_i, E_k)}{\lambda_{int}^{(k)}(E_k)}$$

$$- \frac{\phi_h(E_i)}{\lambda_{int}^{(h)}(E_i)}$$

Decay matrix



typical size

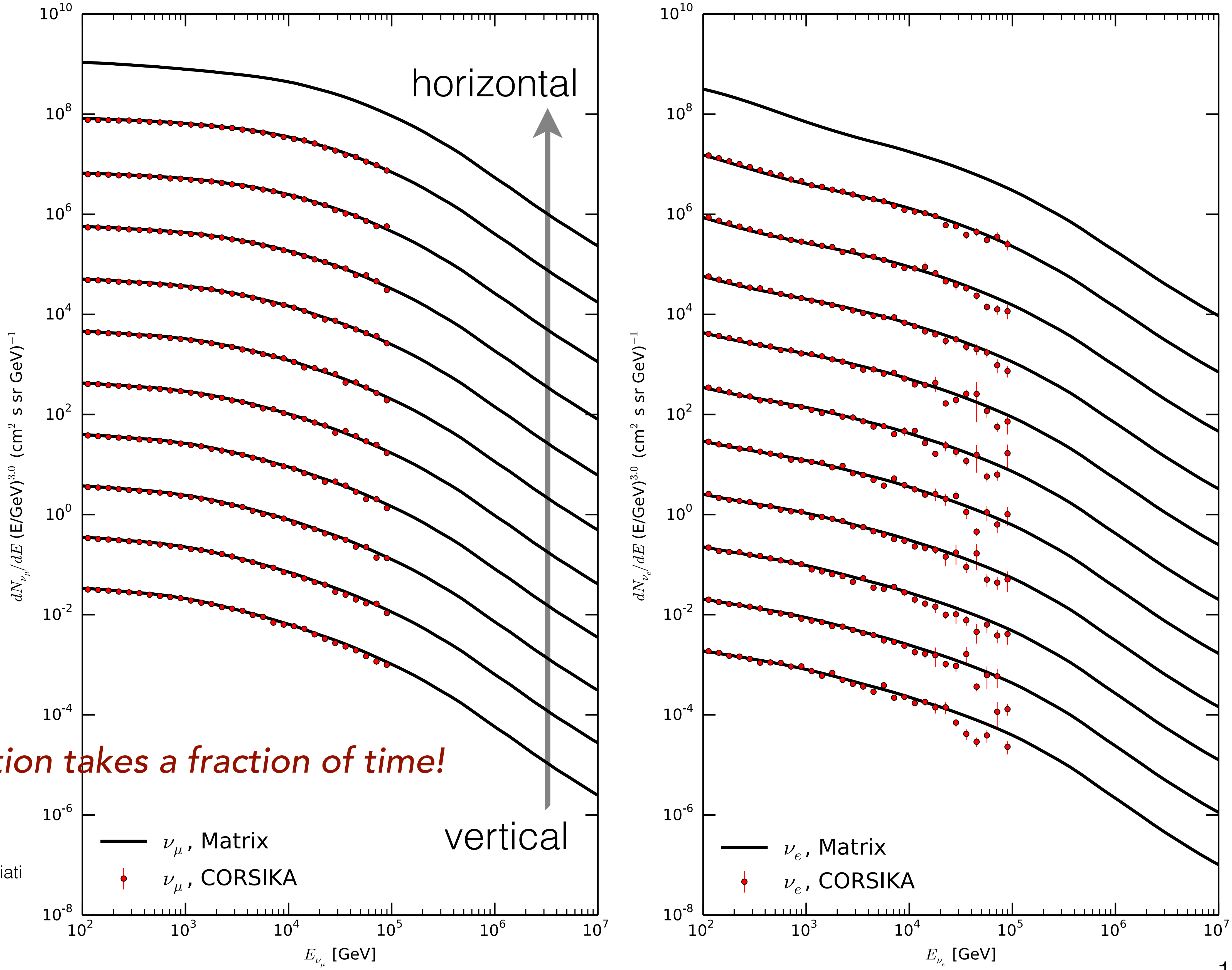
6000 x 6000

few non-zero elements

sparse matrix

Comparison with CORSIKA calculation

Comparison for QGSJET-II-03 + H3a. Offset of 10 between lines.

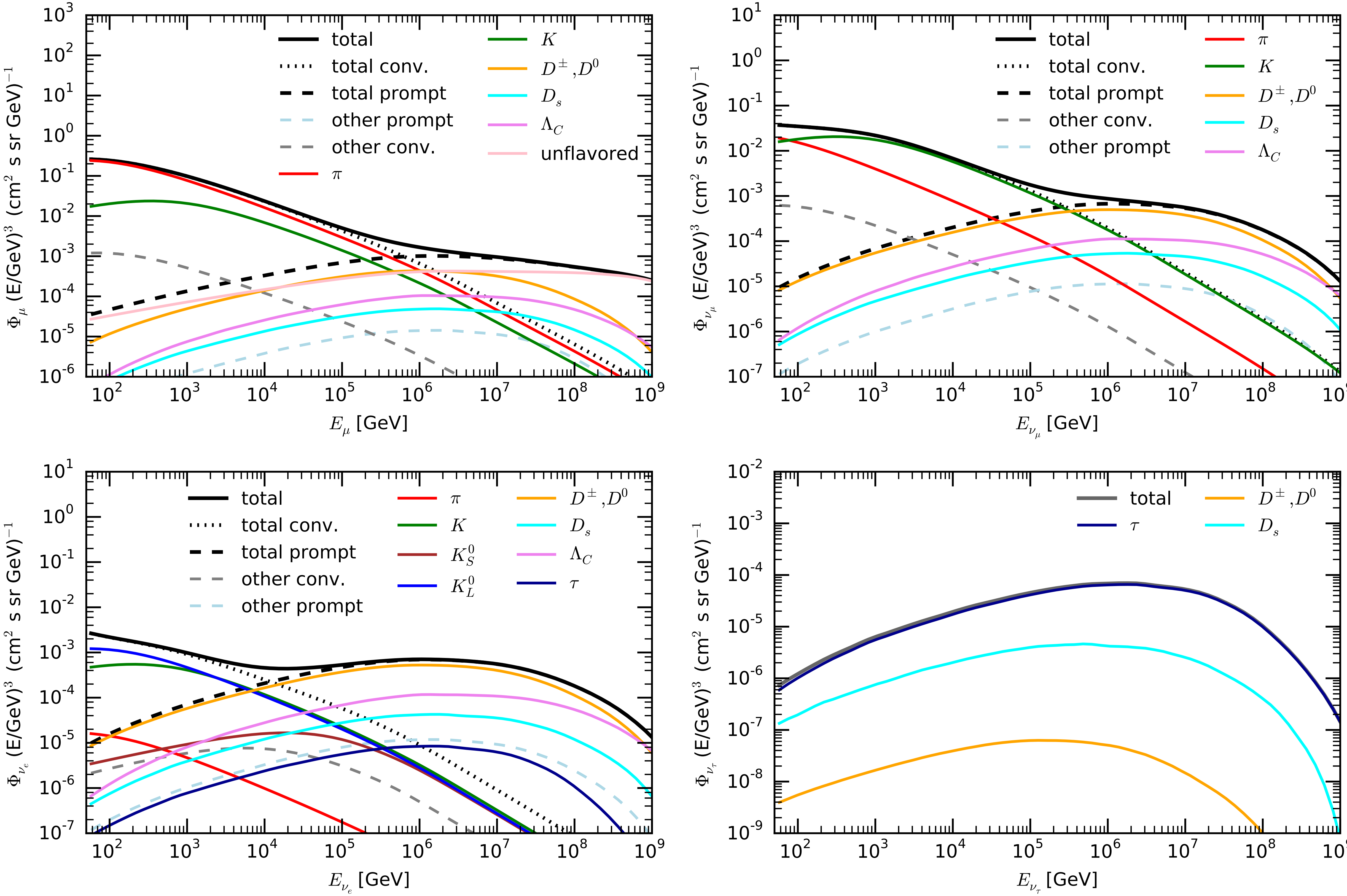


Matrix calculation takes a fraction of time!

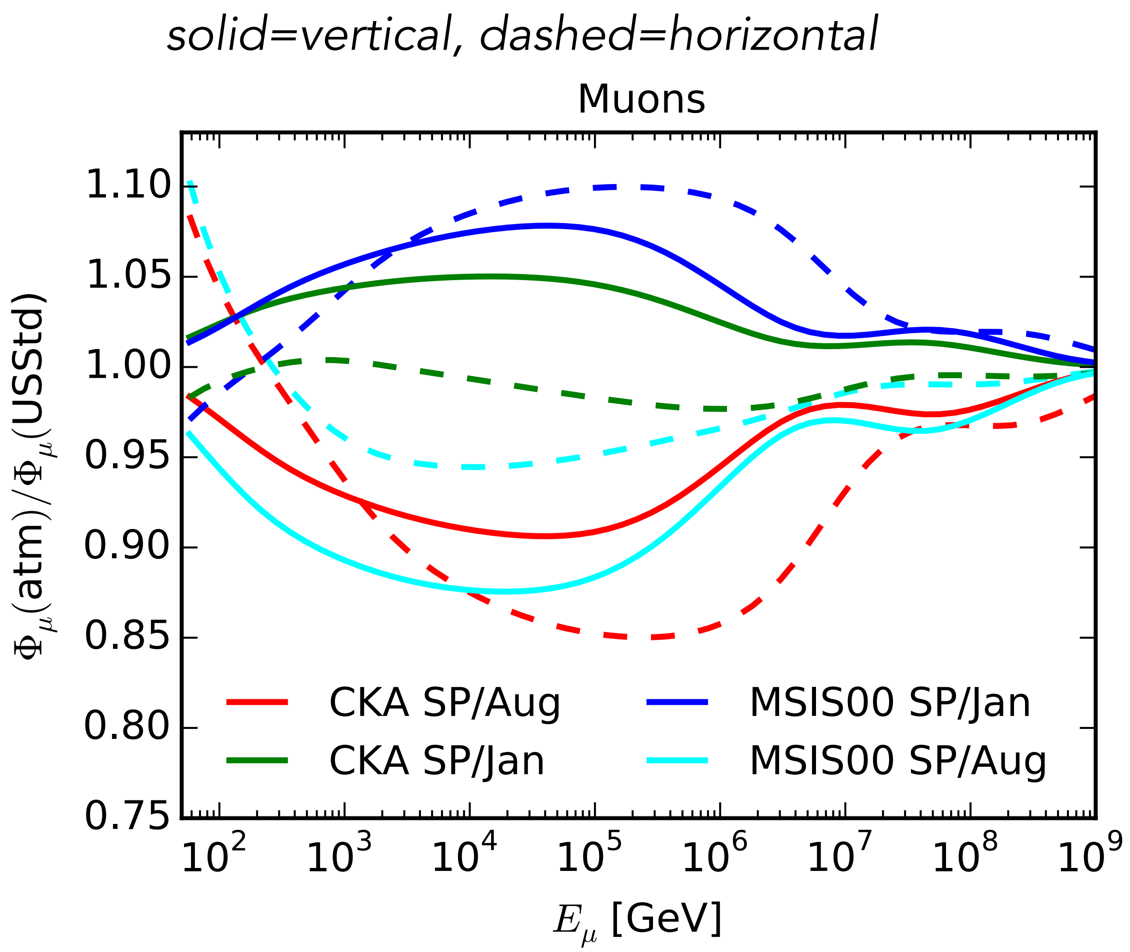
CORSIKA:
Fedynitch, Becker Tjus, Desiati
Phys. Rev. D 86, 114024

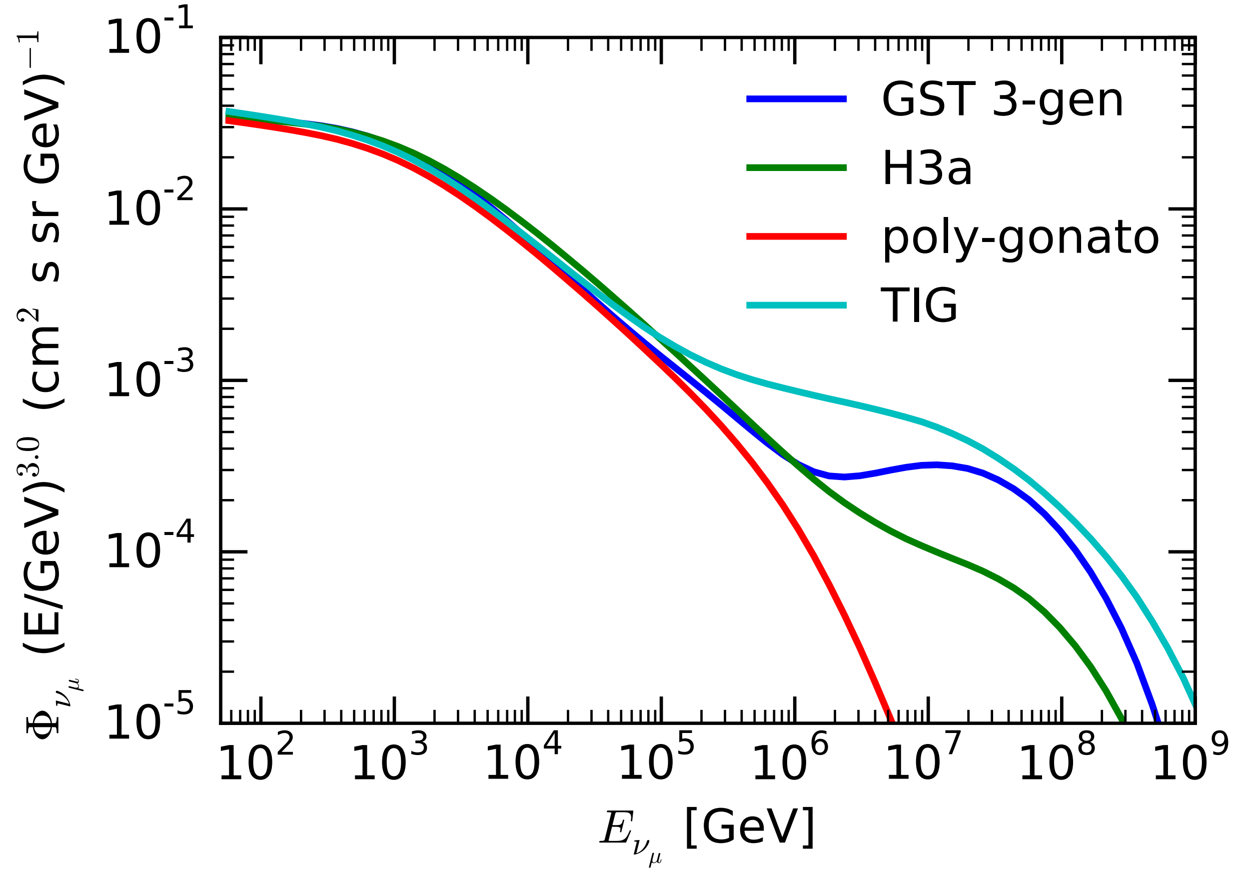
Flux break-down

SIBYLL2.3_rc1 atmospheric lepton fluxes, TIG primary flux model.



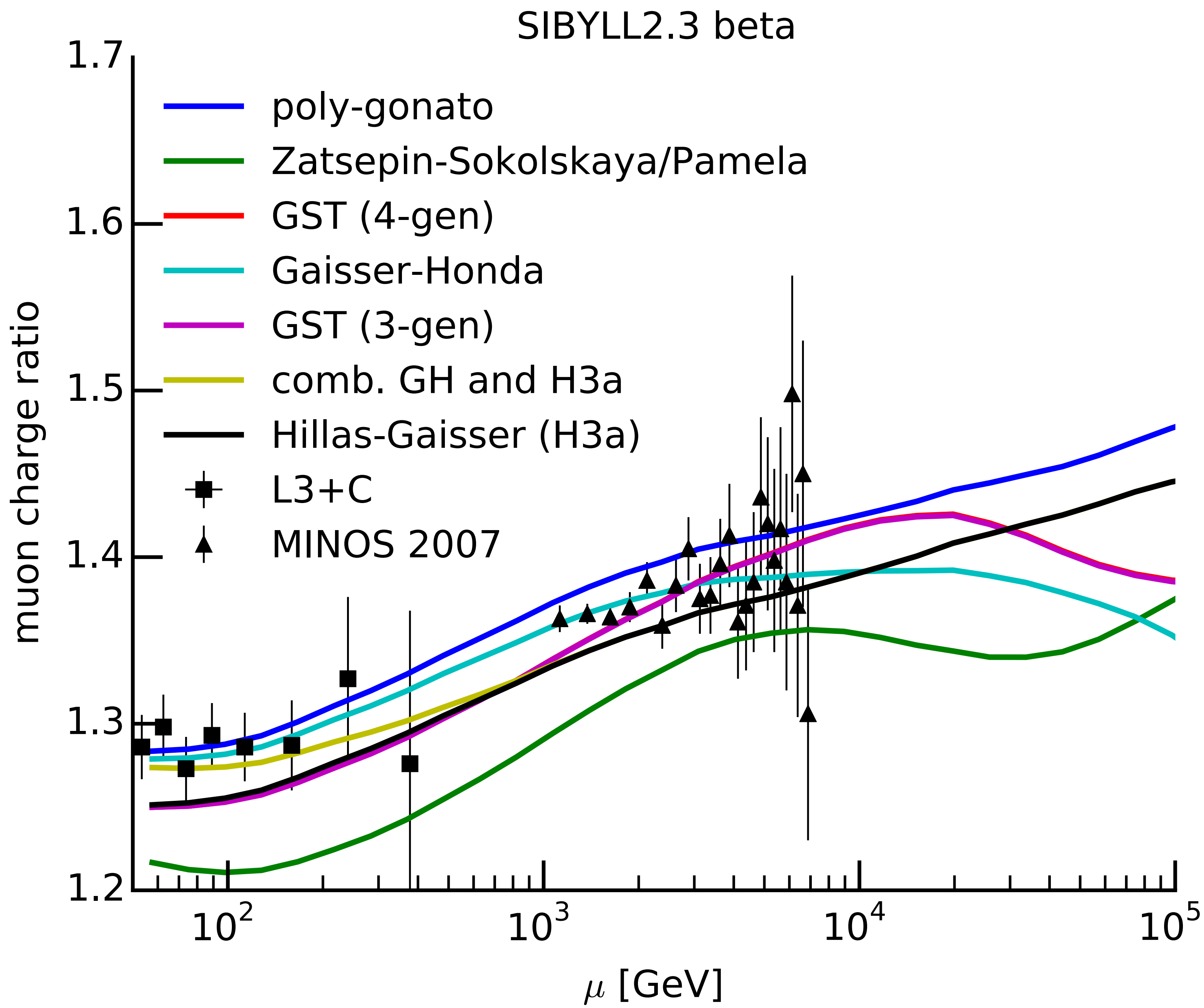
Variation of primary CR flux or atmosphere





- arbitrary all-nucleon flux parameterizations (superposition)
- arbitrary atmospheric/density profiles

Muon charge ratio



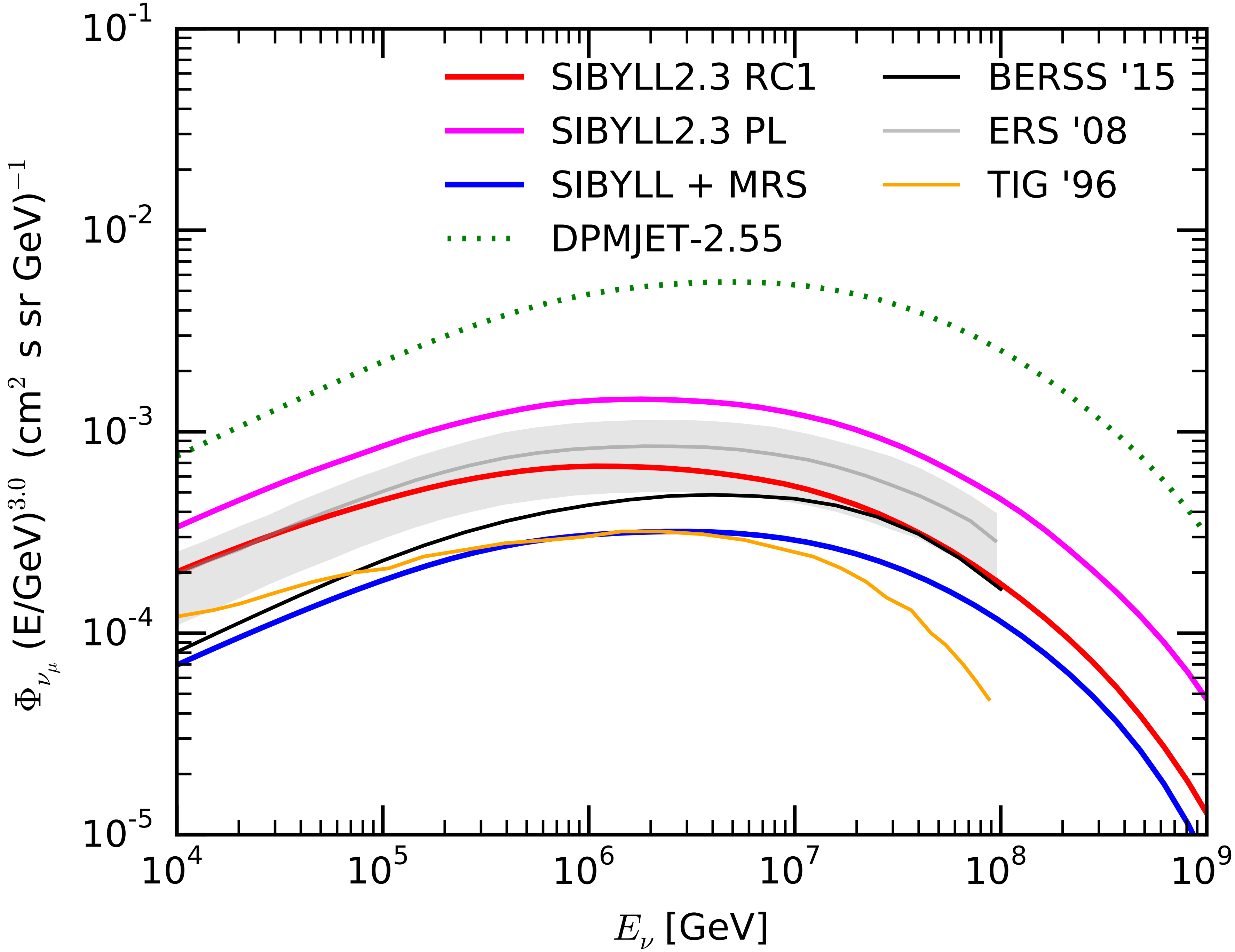
Caution: ambiguity between primary and interaction model!

L3 Collaboration, Physics Letters B 598, 15 (2004)

MINOS Collaboration, Phys. Rev. D 76, 52003 (2007)

For model references see: https://github.com/afedynitch/CRFluxModels

Prompt flux



BERSS: A. Bhattacharya, R. Enberg, M.H. Reno, I. Sarcevic and A. Stasto, arXiv:1502.01076

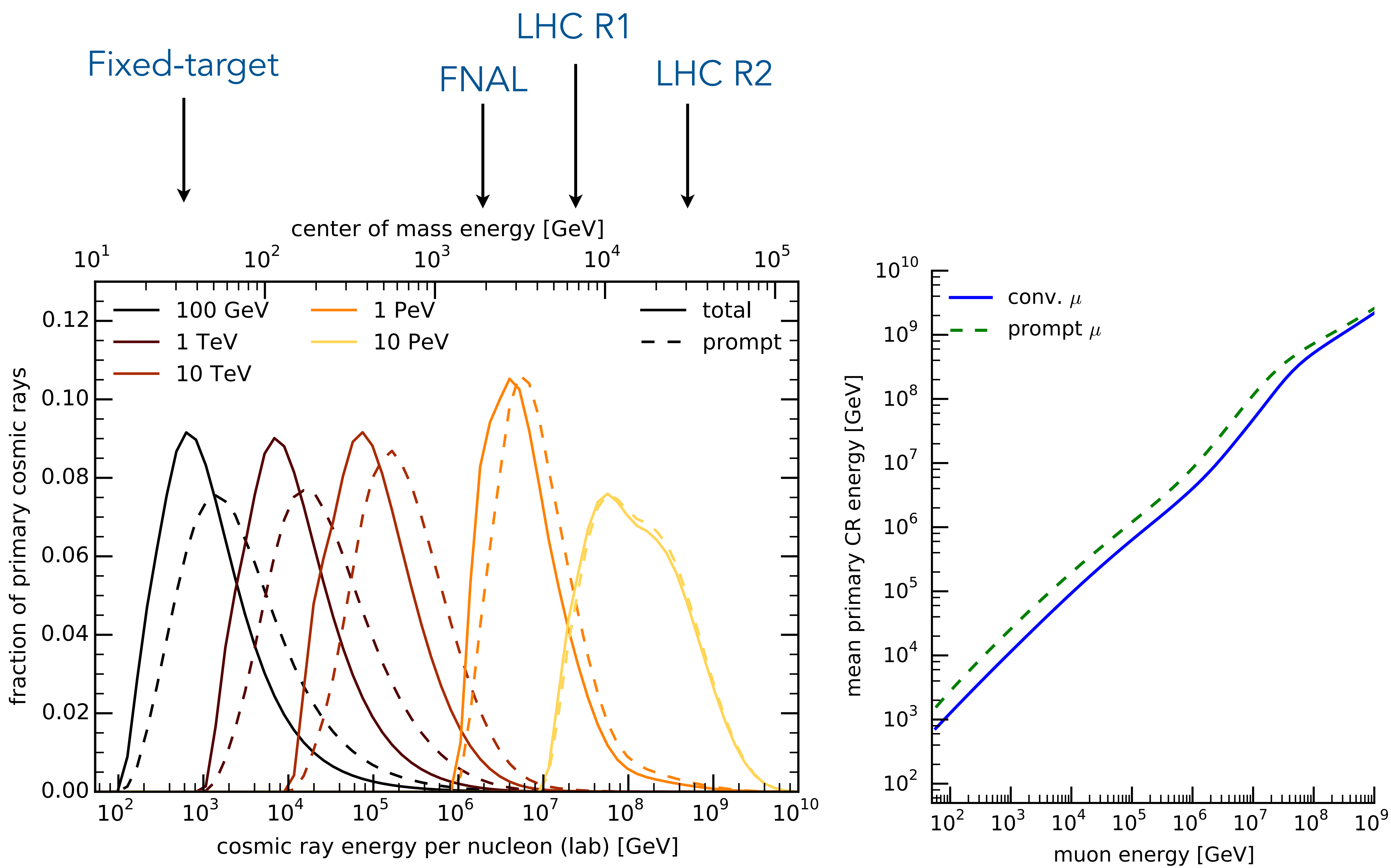
ERS: R. Enberg, M. H. Reno, and I. Sarcevic, Phys. Rev. D 78, 43005 (2008).

*MRS: A. D. Martin, M. G. Ryskin, and A. M. Stasto, Acta Physica Polonica B **34**, 3273 (2003).*

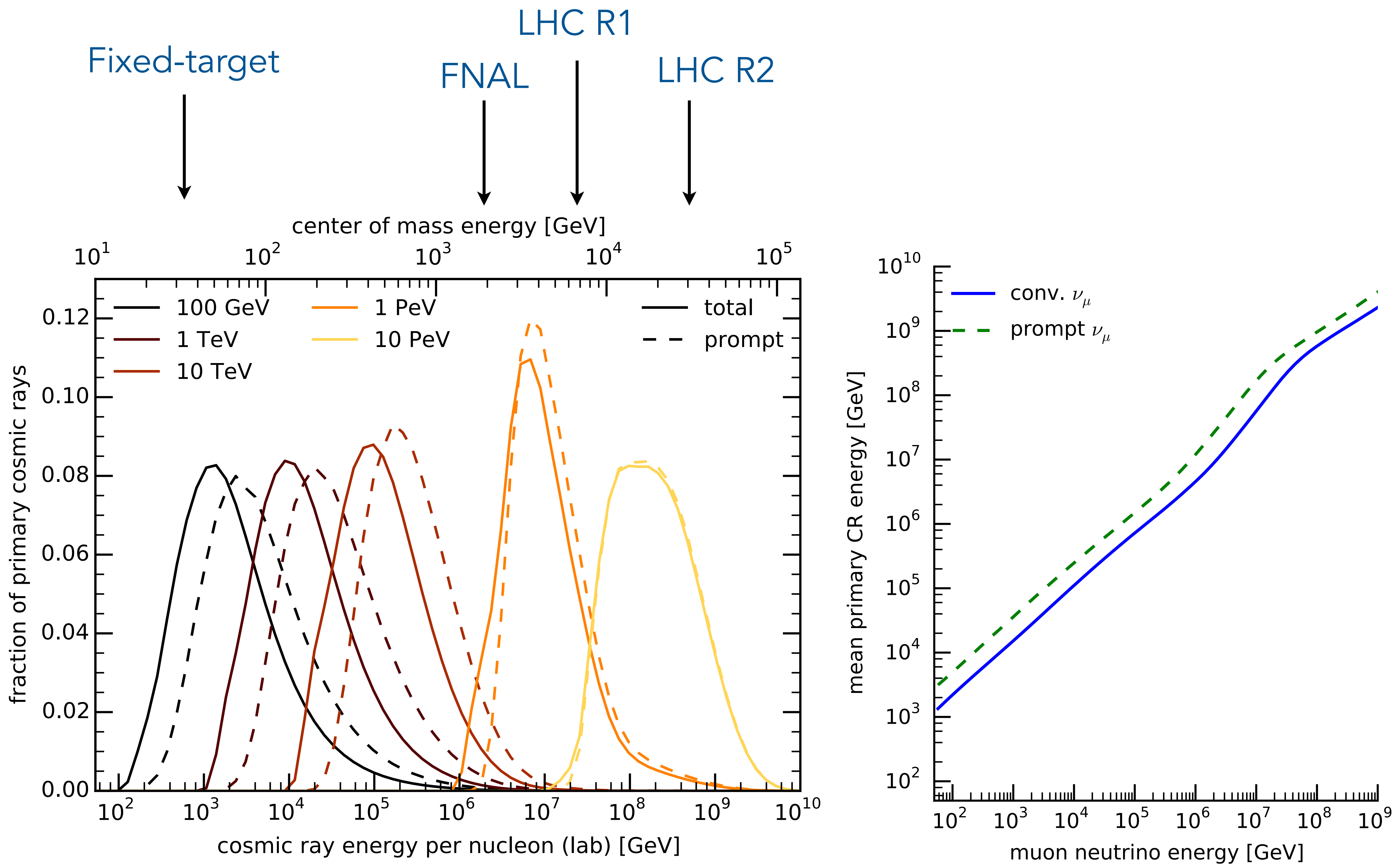
SIBYLL: arXiv:1503.00544 and arXiv:1502.06353

TIG: M. Thunman, G. Ingelman, and P. Gondolo, Astroparticle Physics 5, 309 (1996).

Primary cosmic ray energy (muons)

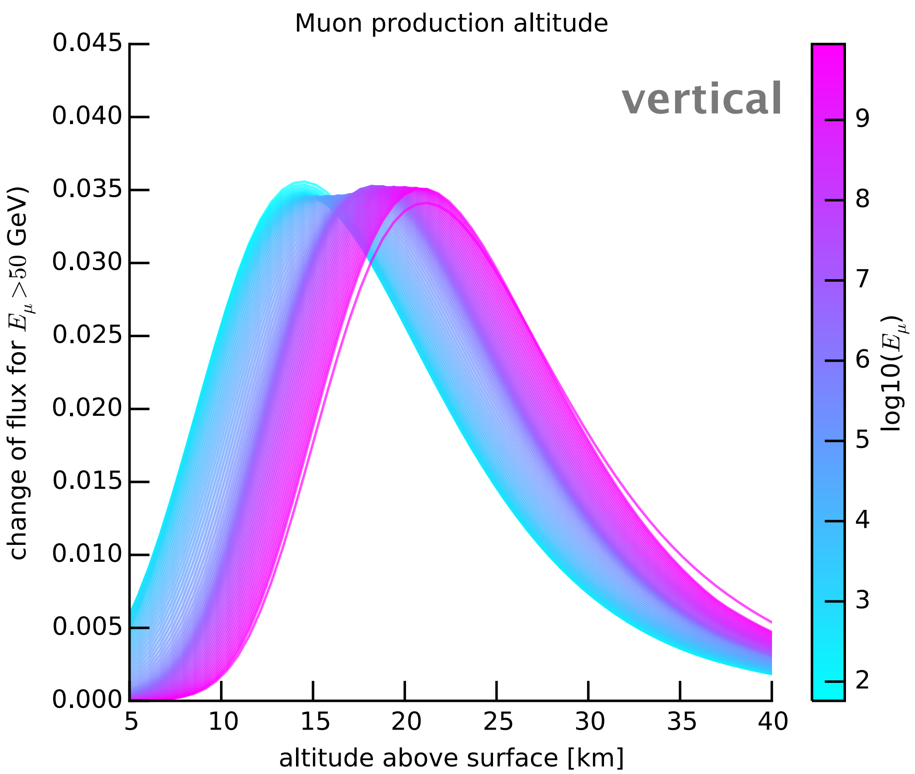


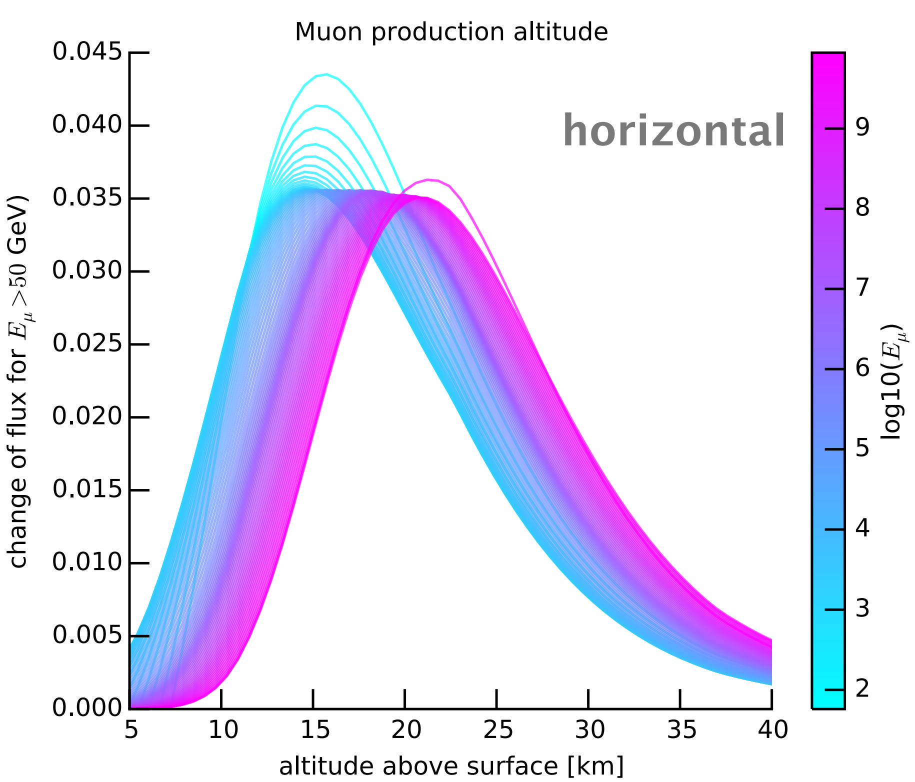
Primary cosmic ray energy (muon neutrinos)



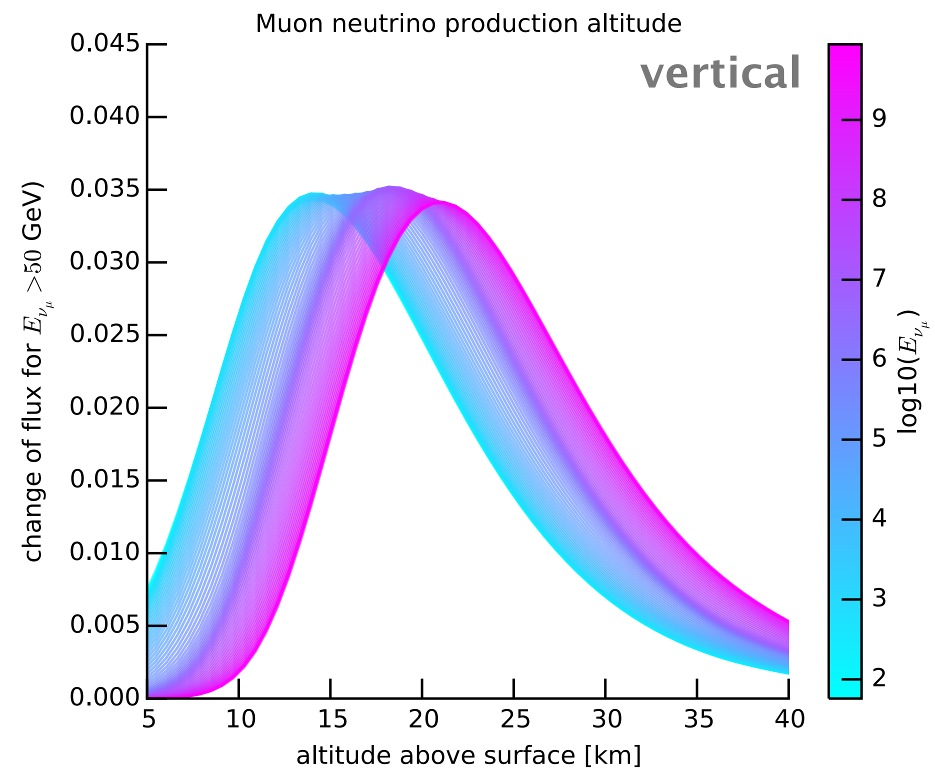
Production height

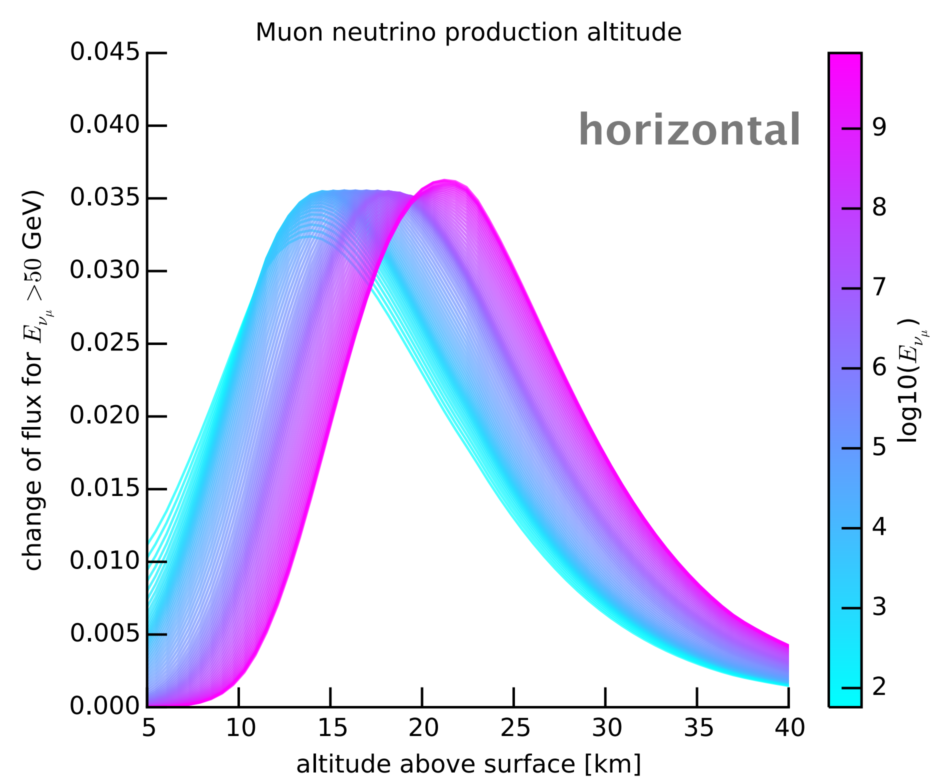
Muons





Muon neutrinos





Summary/Outlook

SIBYLL 2.3

- Experiments with internal physical models are still ongoing
- final tuning can only be done after the physics contents are fixed
- a(n intermediate) version is expected this summer
- majority of distributions are better than in SIBYLL 2.1

MCEq

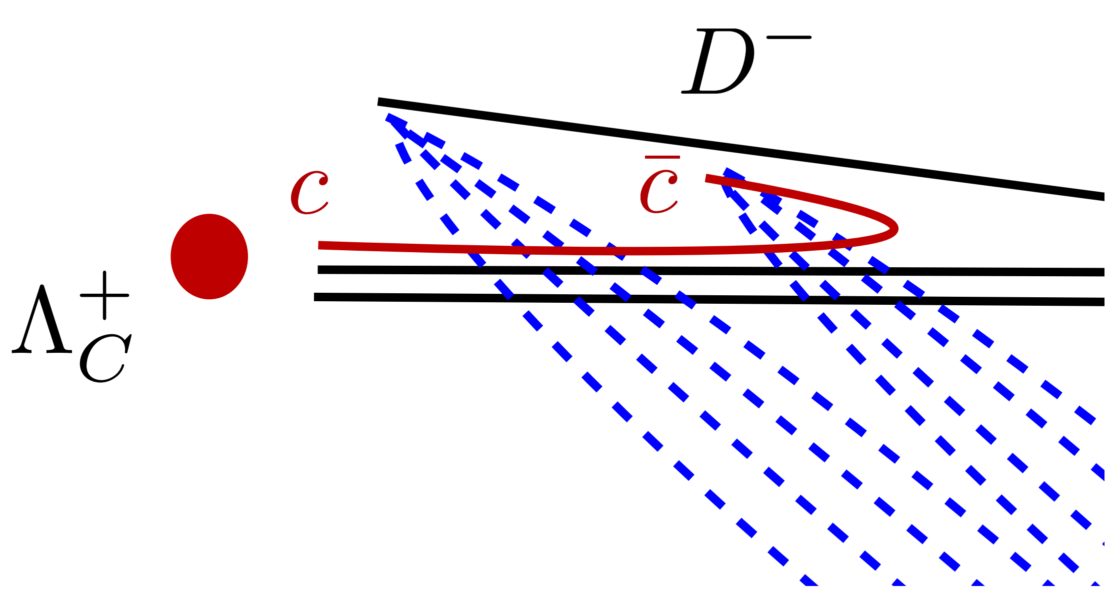
- code near final and available on github: https://github.com/afedynitch/MCEq
- some analysis in IceCube have already experience with using it (Gabriel Collin)
- the precision is limited by the physical inputs: primary flux, hadronic interactions
- approximations in the cascade solution or due to numerics are negligible
- we are working on a full publication

Backup

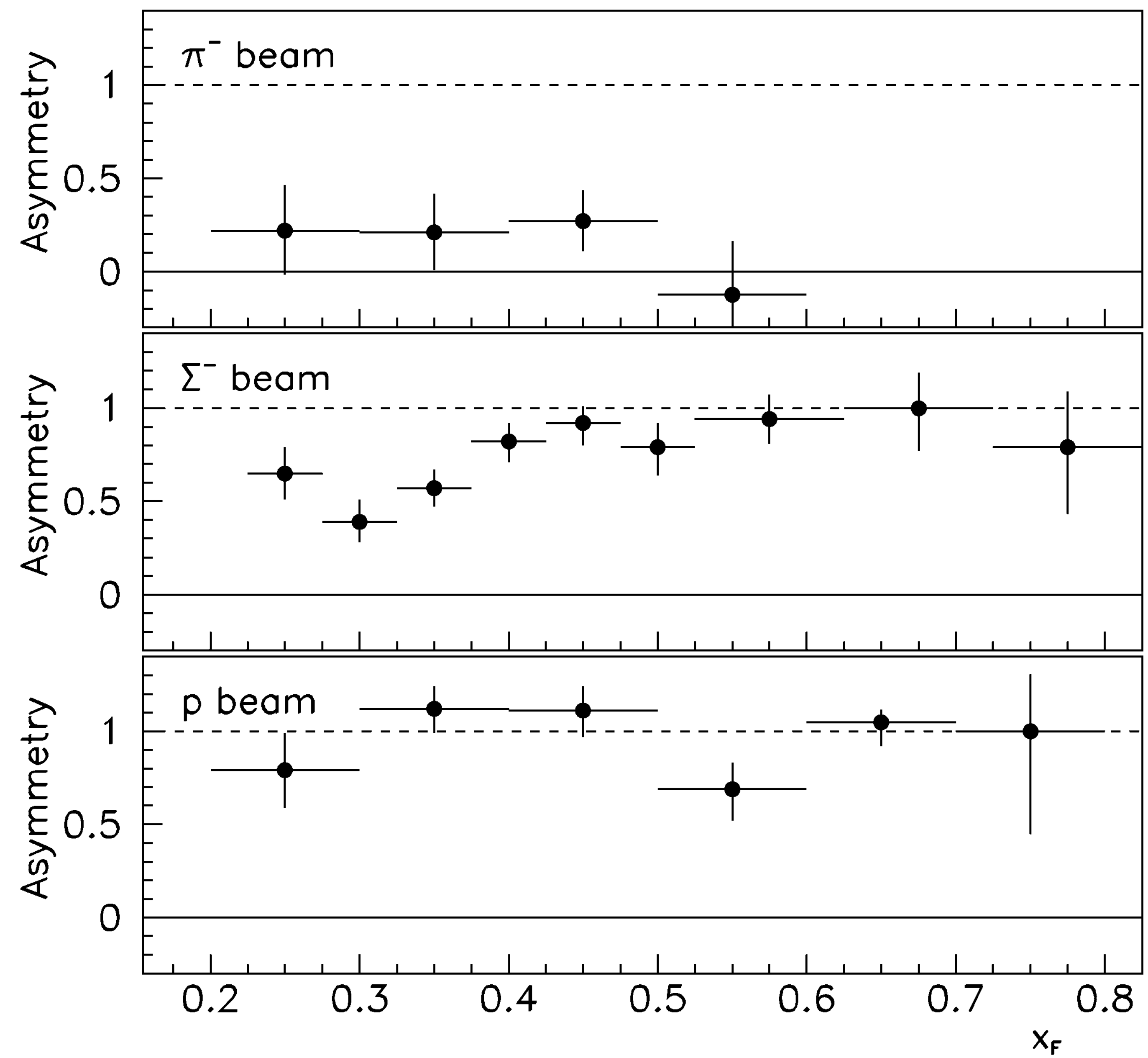
Leading/non-perturbative charm

Asymmetry

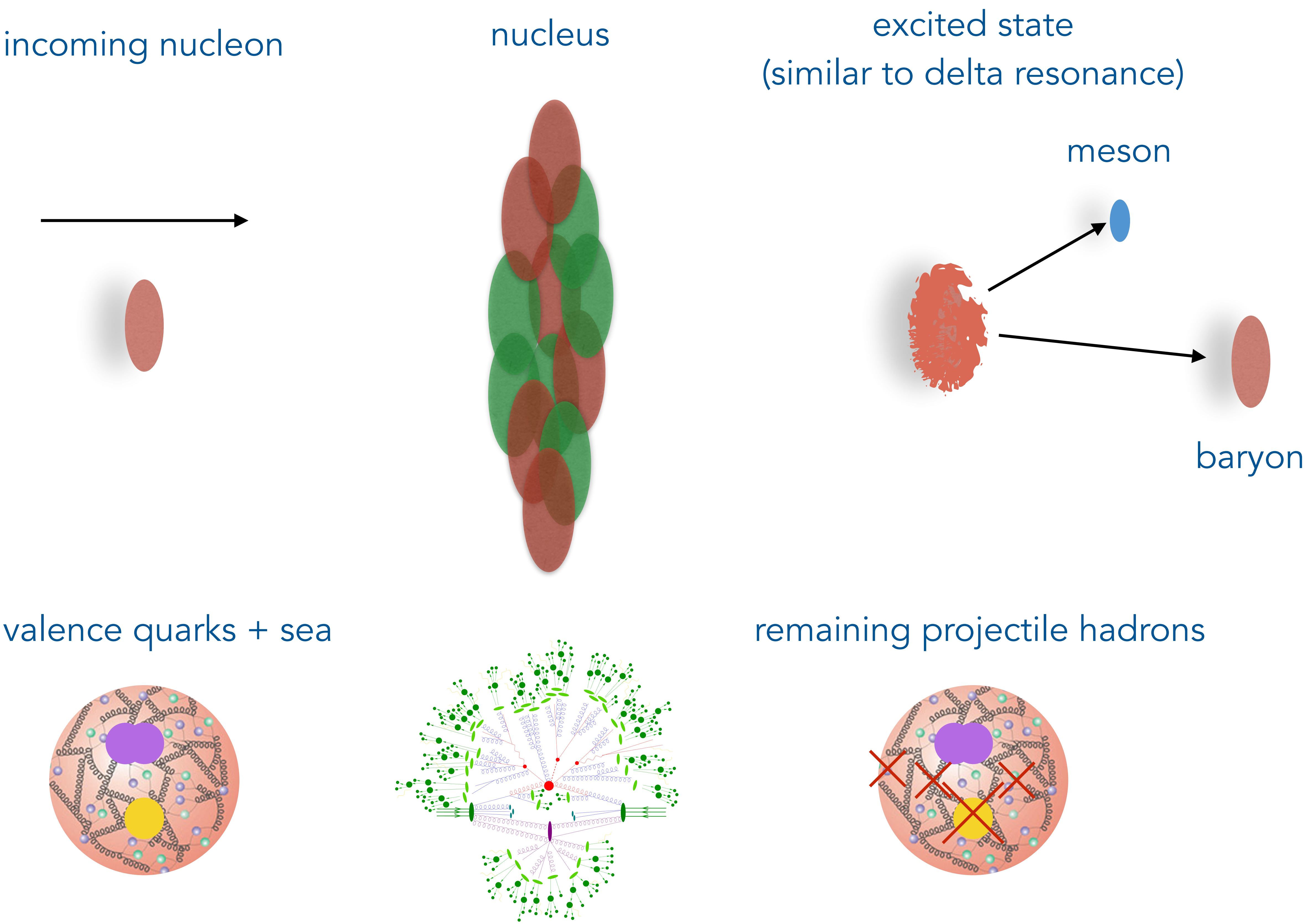
$$A \equiv \frac{\Lambda_C - \bar{\Lambda}_c}{\Lambda_C + \bar{\Lambda}_c}$$



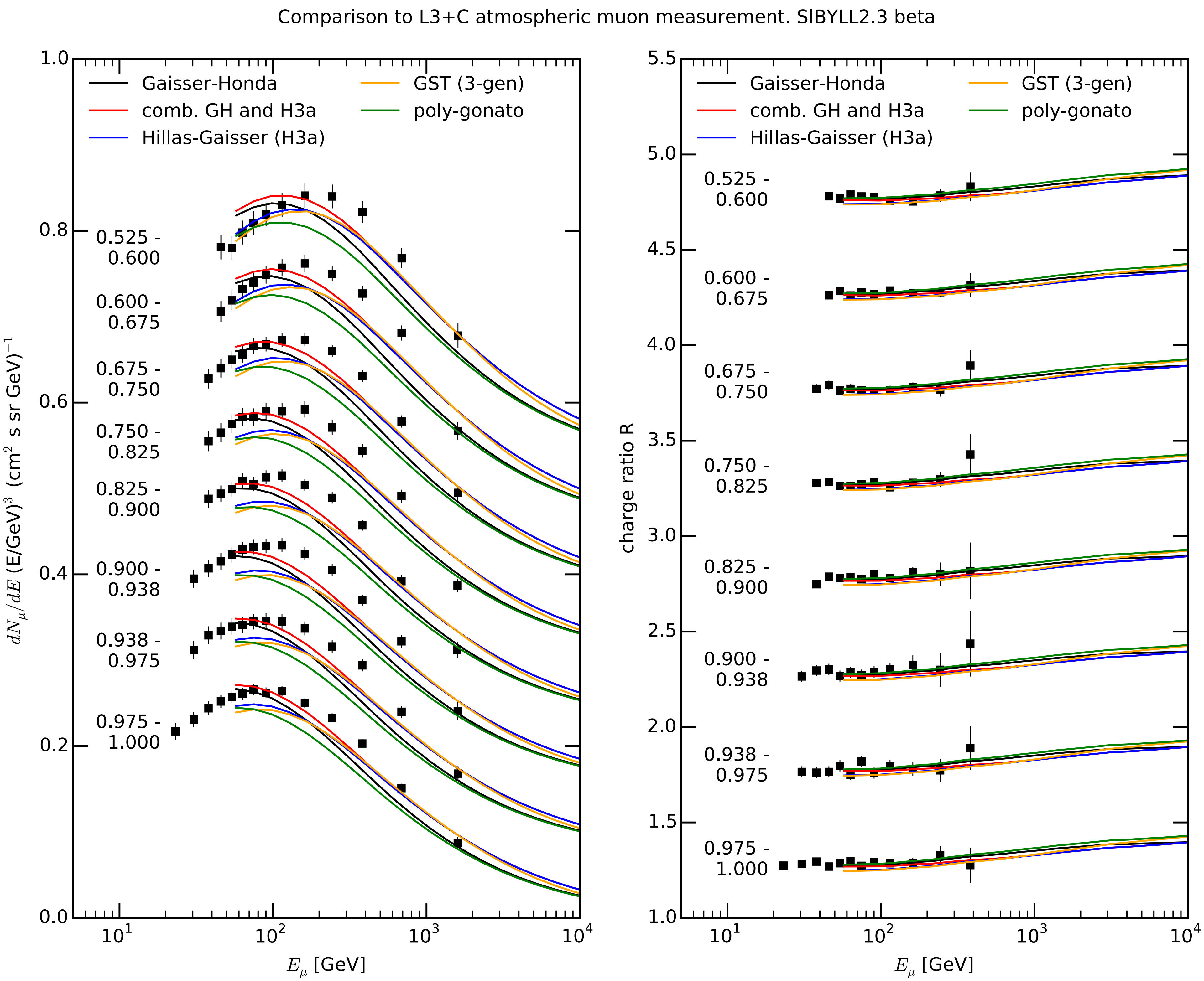
SELEX Collaboration, F. G. Garcia et al., Physics Letters B 528, 49 (2002).



Sketch of remnant model



Comparison with atmospheric muons



Proton cascade

$$\frac{\mathrm{d}\Phi_p(E,X)}{\mathrm{d}X} = -\frac{\Phi_p(E,X)}{\lambda_{int,p}(E)} + \int_E^\infty \frac{\Phi_p(E',X)}{\lambda_{int,p}(E')} \frac{\mathrm{d}N_{p\to p}(E')}{dE'} \mathrm{d}E'$$

Factorization ansatz

$$\Phi_p(E,X) = A(X)E^{-\gamma}$$

$$\frac{\mathrm{d}A(X)}{\mathrm{d}X} = -\frac{A(X)}{\lambda_{int,p}}\left[1 - \int_0^1 x^{\gamma-1}\frac{\mathrm{d}N_{p\to p}}{\mathrm{d}x}\right]$$

$$= -\frac{A(X)}{\lambda_{int,p}}[1 - Z_{pp}] = -\frac{A(X)}{\Lambda}$$

Solution

$$\Phi_p(E,X) = A(0)e^{-X/\Lambda}E^{-\gamma}$$

Problems

- proton production properties (Z_{pp}) independent of energy (scaling)
- interaction cross-sections independent of energy
- valid only for power-law primary spectra

Couplings in a (p,n,pion) system

Coupled cascade equation for pions

$$\frac{\mathrm{d}\Phi_\pi}{\mathrm{d}X} = \boxed{-\frac{\Phi_\pi}{\lambda_{int,p}} - \frac{\Phi_\pi}{\lambda_{dec,p}}} + \boxed{Z_{\pi\pi}\frac{\Phi_\pi}{\lambda_{int,\pi}} + \underbrace{\boxed{Z_{p\pi}\frac{\Phi_p}{\lambda_{int,p}} + Z_{n\pi}\frac{\Phi_n}{\lambda_{int,n}}}}_{\text{coupling}}}$$

\+ decays into pions

coupled cascade equation for muons

$$\frac{\mathrm{d}\Phi_\mu}{\mathrm{d}X} = -\frac{\Phi_\mu}{\lambda_{dec,\mu}} + S(\pi \to \mu) = \boxed{-\frac{\Phi_\mu}{\lambda_{dec,\mu}}} + \boxed{Z^D_{\pi\to\mu}\frac{\Phi_\pi}{\lambda_{dec,\pi}}}$$

and muon neutrinos

$$\frac{\mathrm{d}\Phi_\nu}{\mathrm{d}X} = S(\pi \to \nu)$$

Becomes complicated if more channels/particles are included

Important lepton production channels

conventional

$$p, A + \mathrm{air} \rightarrow \pi^{\pm}, \pi^0, \mathrm{K}^{\pm}, \mathrm{K}^0_{\mathrm{S,L}}$$

muons and muon neutrinos

$$\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu})$$

electron neutrinos

$$K^{\pm}, K^0_L \rightarrow [\pi^{\pm}, \pi^0]e^{\pm}\nu_e(\bar{\nu}_e)$$

prompt

$$p, A + \mathrm{air} \rightarrow \mathrm{D}, \Lambda_{\mathrm{C}} \rightarrow \nu_{\mu}, \nu_{\mathrm{e}}, \mu$$

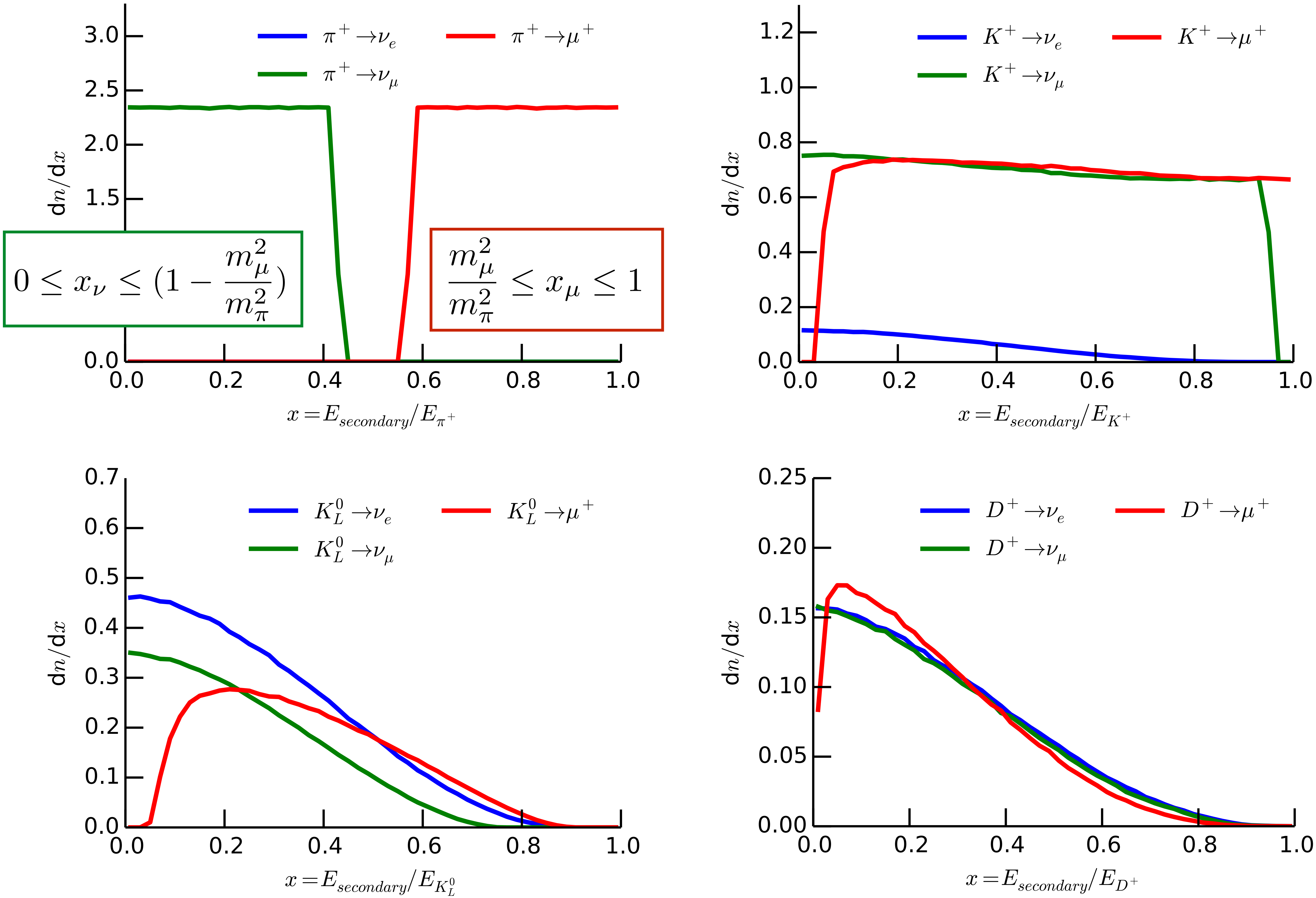
Subset of dominant decay channels

decay channel	branching ratio (BR)
$\mu^- \rightarrow e^-\bar{\nu}_e\nu_{\mu}$	100 %
$\pi^+ \rightarrow \mu^+\nu_{\mu}$	99.9877 %
$K^0_{e3} : K^0_L \rightarrow \pi^{\pm}e^{\mp}\nu_e$	40.55 %
$K^0_{\mu 3} : K^0_L \rightarrow \pi^{\pm}\mu^{\mp}\nu_{\mu}$	27.04 %
$K^+ \rightarrow \mu^+\nu_{\mu}$	63.55 %
$K^+_{e3} : K^+ \rightarrow \pi^0 e^+\nu_e$	5.07 %
$K^+_{\mu 3} : K^+ \rightarrow \pi^0\mu^+\nu_{\mu}$	3.353 %
$D^+ \rightarrow \overline{K}^0\mu^+\nu_{\mu}$	9.2 %
$D^0 \rightarrow K^-\mu^+\nu_{\mu}$	3.3 %

+ *charge conjugates*

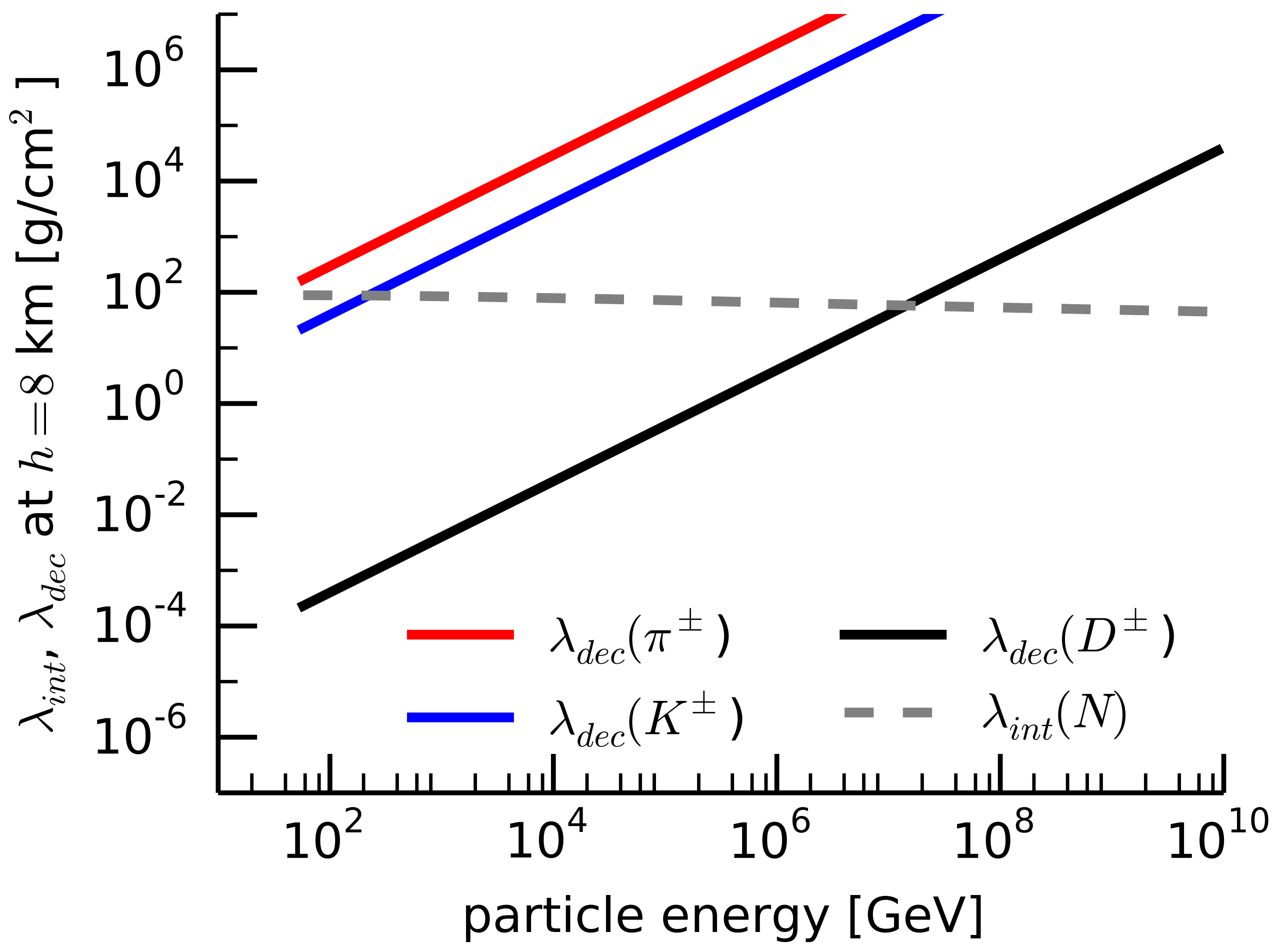
http://pdg.lbl.gov

Energy distribution in decays



Numerical challenges

For pion and kaon decay is a slow process



For short-lived particles

$$\lambda_{dec}^D \ll \lambda_{int} \longrightarrow \Delta X \propto \lambda_{dec}^D$$

Interaction length

$$\lambda_{int,h}(E) = \frac{m_{Air}}{\sigma_{h-Air}^{inel}(E)}$$

Independent of X or height

weak dependence on energy

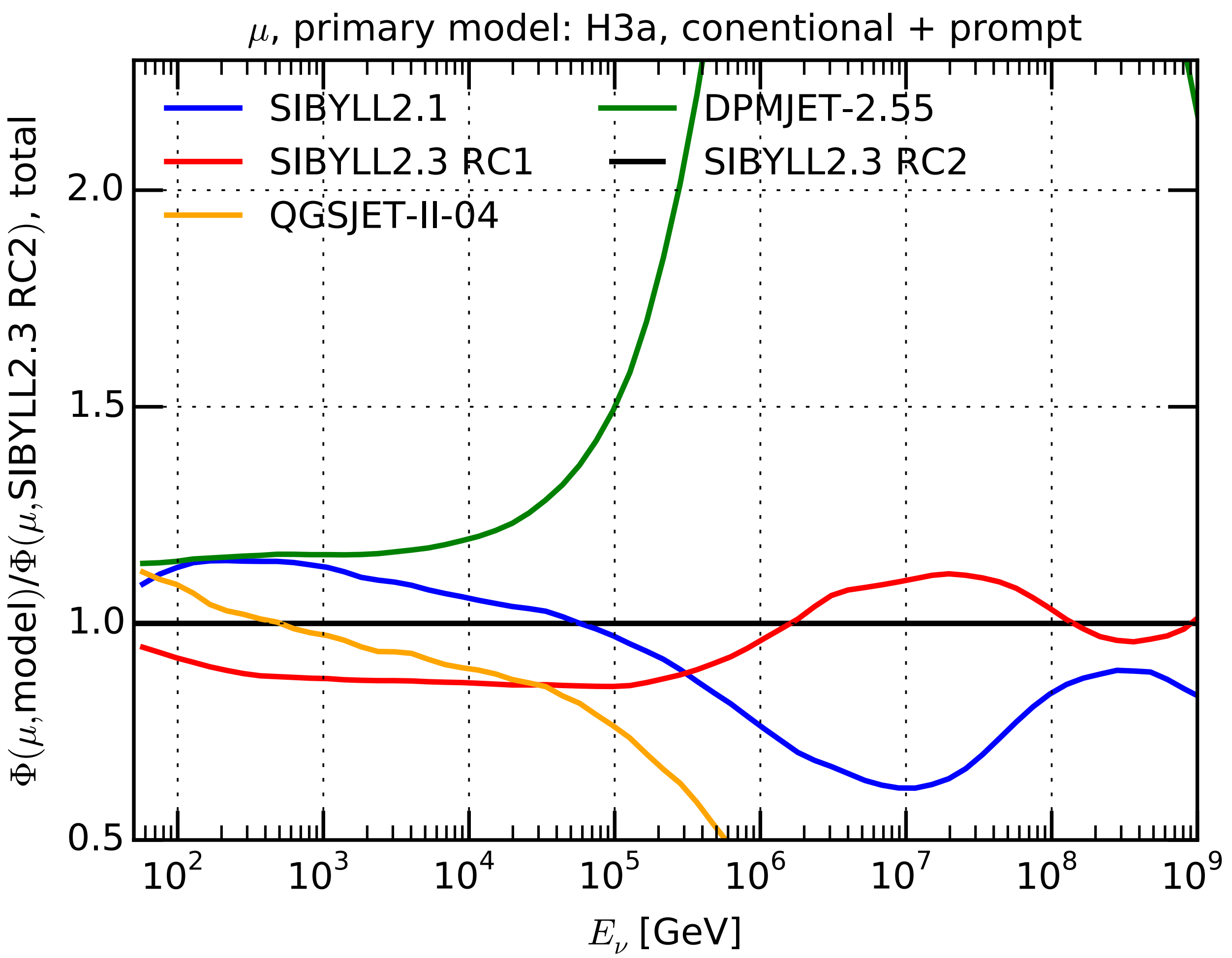
Decay length

$$\lambda_{dec,h}(E,X) = \frac{c\tau_h E \rho_{Air}(X)}{m_h c^2} = \frac{E X \cos\theta}{E_{crit,h}}$$

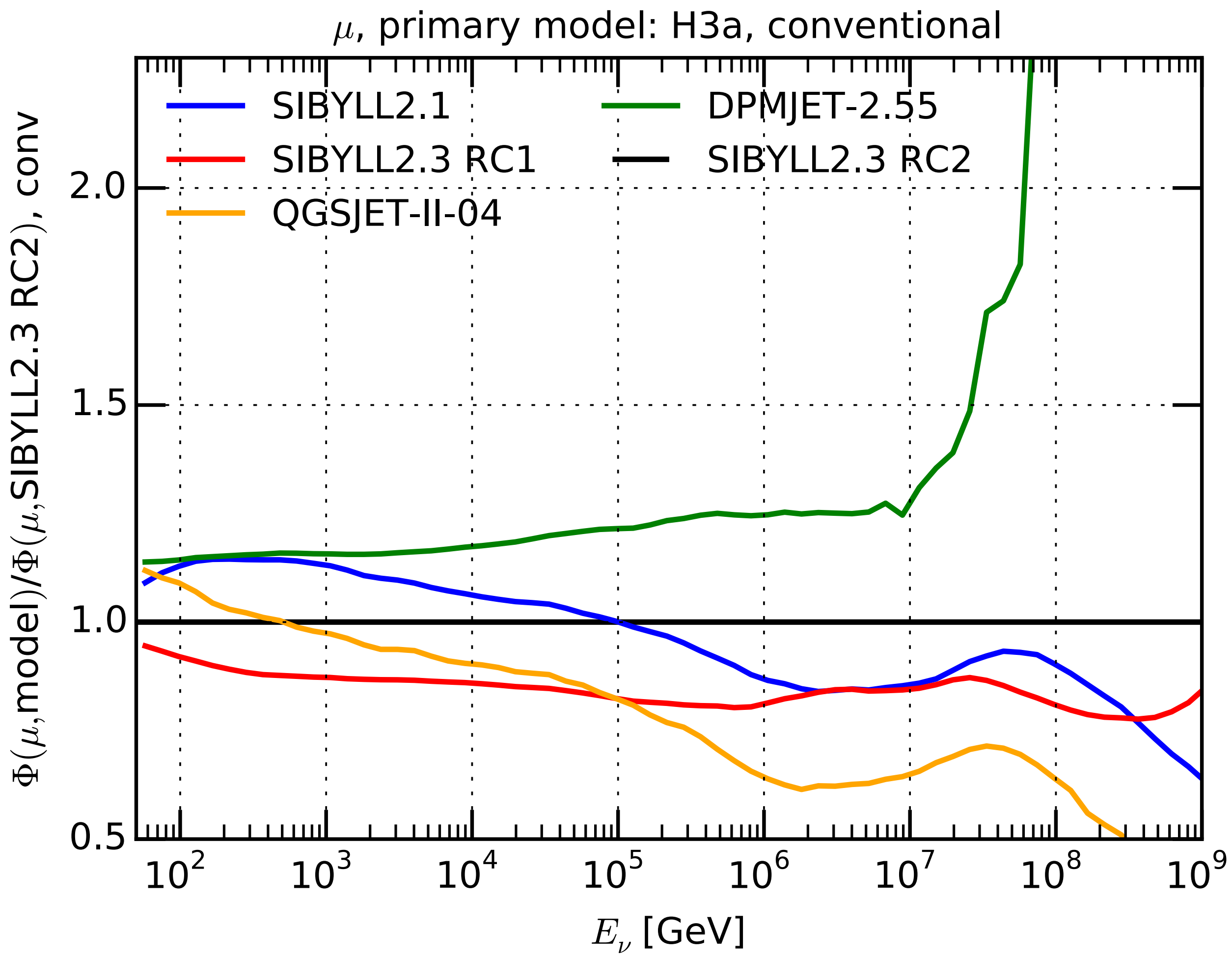
proportional to energy (boost)

Relative difference between models - muons

total

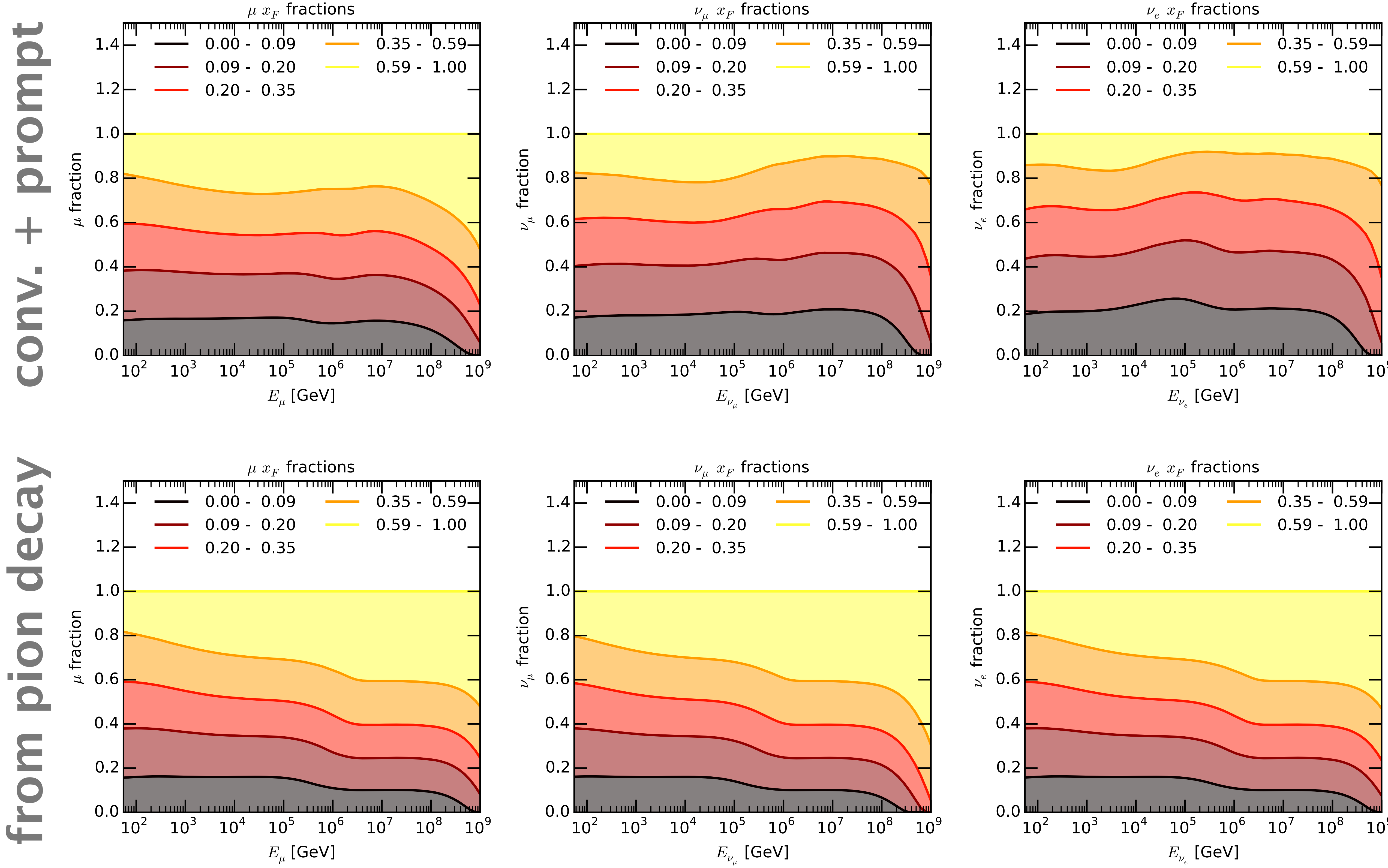


conventional

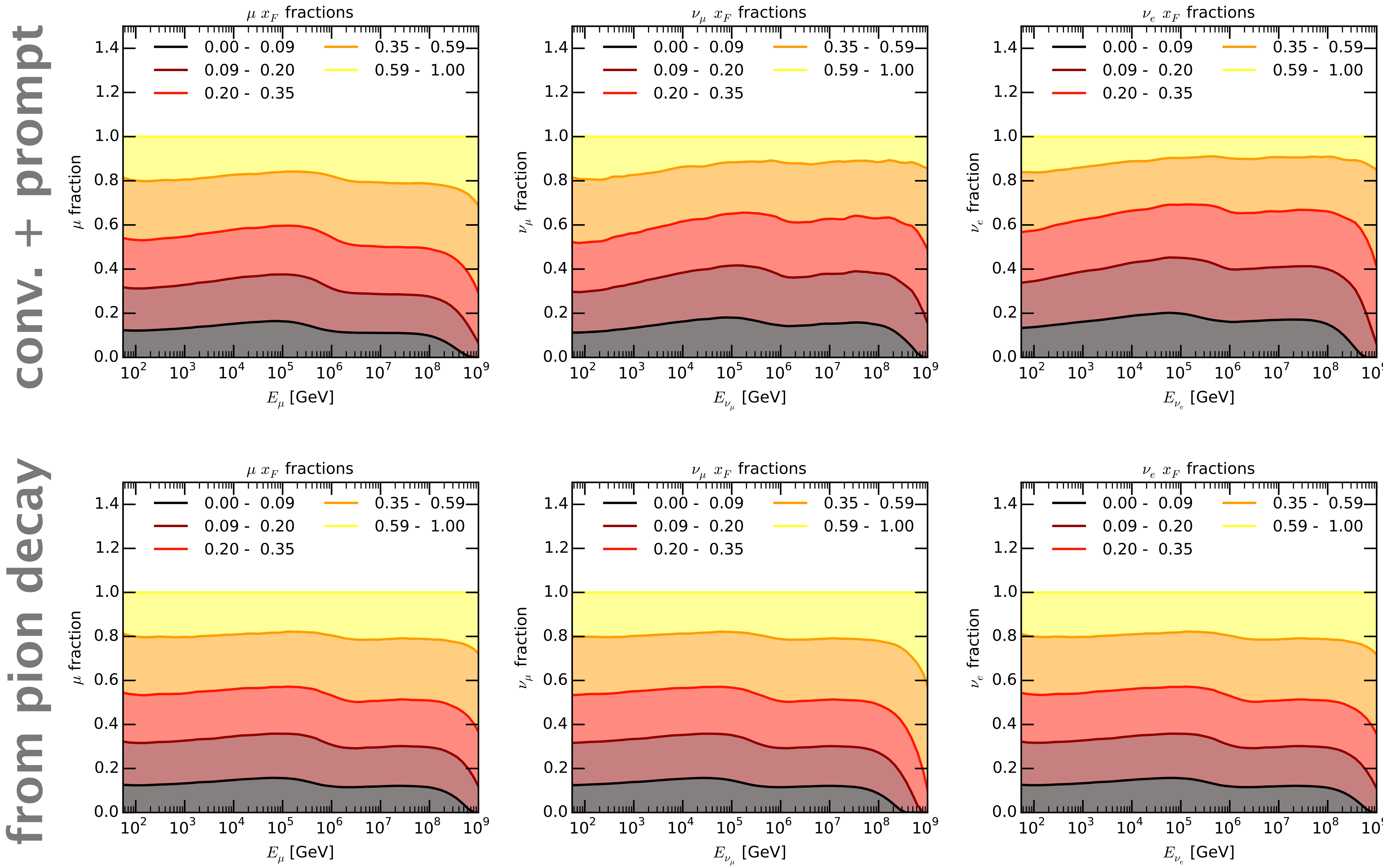


Flux for a each interaction model, divided by flux calculated with current dev. version SIBYLL 2.3

Feynman x range - SIBYLL 2.3 RC1



Feynman x range - SIBYLL 2.1



Feynman x range - QGSJET-II-04