

# Results from the T2K long baseline neutrino oscillation experiment

IceCube Particle Astrophysics Symposium

Casey Bojechko for the T2K collaboration

University of Victoria

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University  
of Victoria



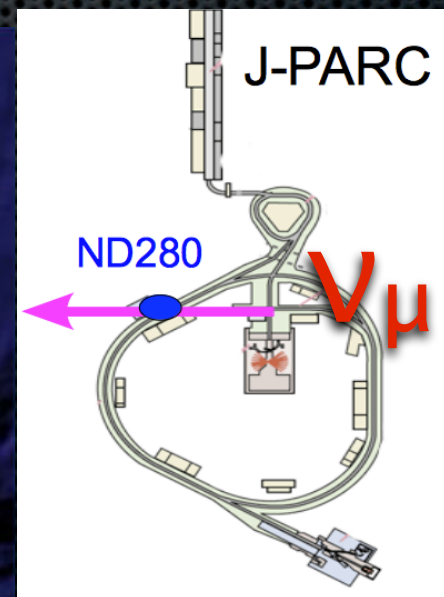
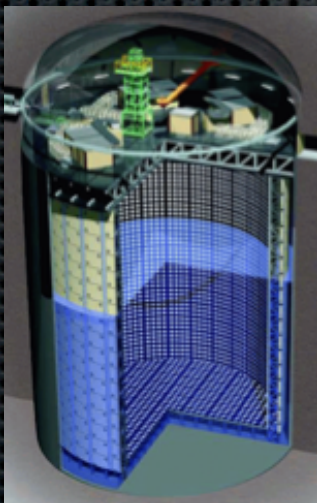
# Outline

- ✧ T2K Experiment: Elements and physics goals.
- ✧ Oscillation Analyses
  - ✧  $\nu_e$  appearance
  - ✧  $\nu_\mu$  disappearance
- ✧ Future prospects



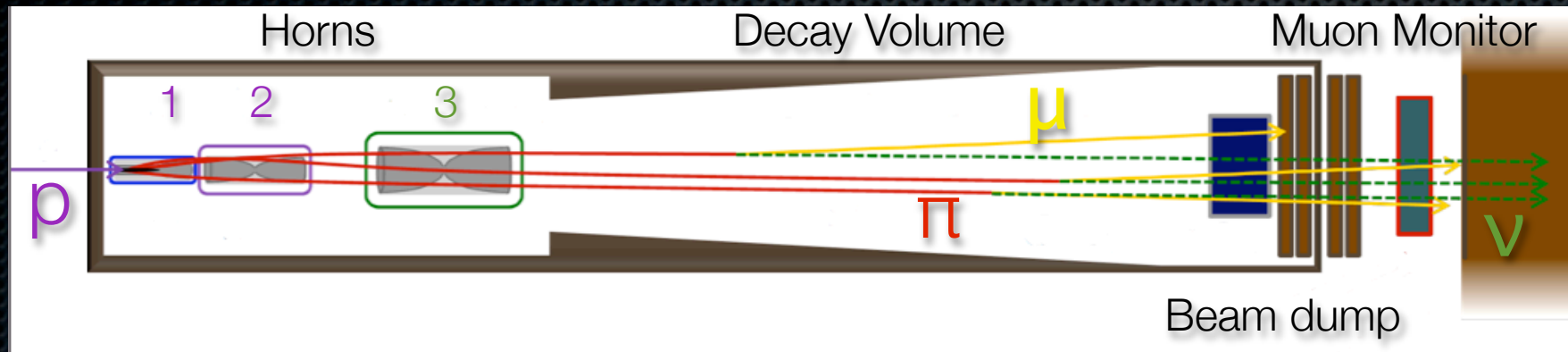
# T2K(Tokai to Kamioka)

- Long baseline neutrino oscillation experiment
  - Measurement of neutrino oscillation between near detector (J-PARC) and Super-Kamiokande.
- Main Physics Goals
  - Search for  $\nu_e$  **appearance**  $\nu_\mu \rightarrow \nu_e$
  - Precise measurement of  $\Delta m^2_{32}$ ,  $\theta_{23}$ .  $\nu_\mu$  **disappearance**  $\nu_\mu \rightarrow \nu_x$





# Neutrino Beam



- ✦ 30 GeV protons hit graphite target
- ✦ Pions produced in proton interactions on a target focused by 3 magnetic horns

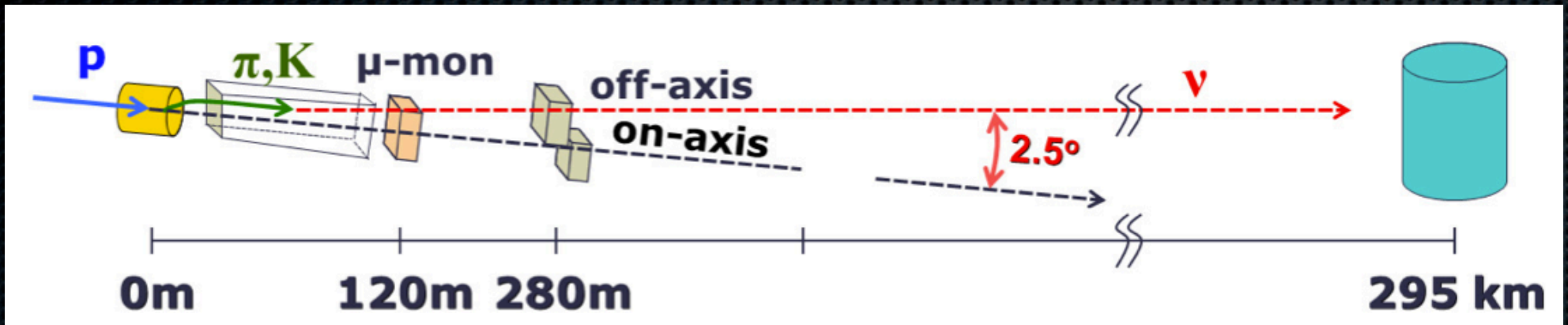
- ✦ focus  $\pi^+$ , defocus  $\pi^-$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

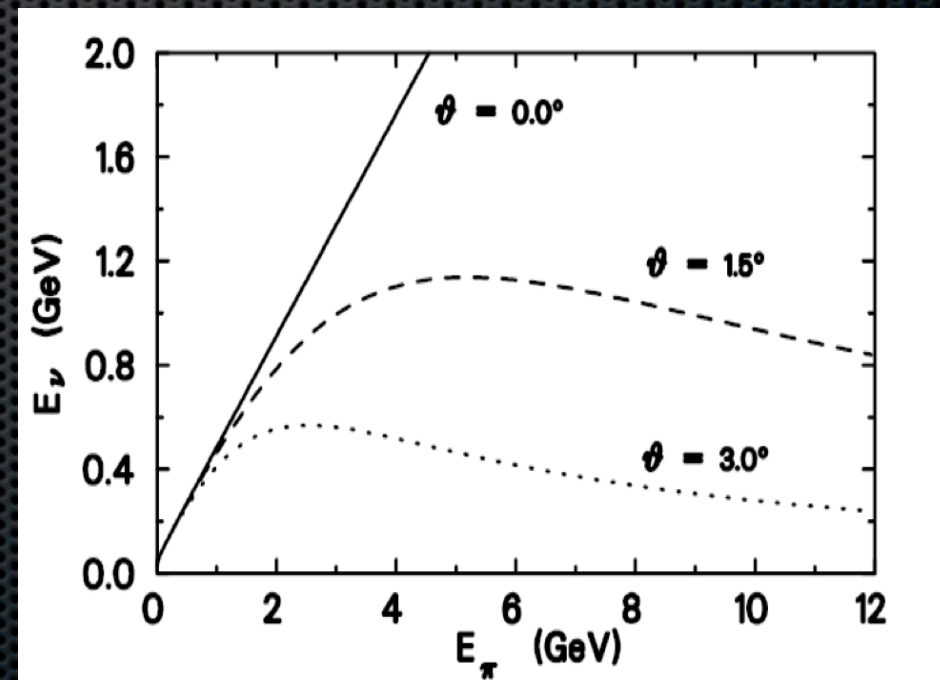
- ✦  $\mu$  monitor at far end of beam dump
- ✦ Creates  $\nu_\mu$  pure beam
- ✦  $\bar{\nu}_\mu$  and  $\nu_e$  are  $\sim$  few percent



# Off-Axis Beam

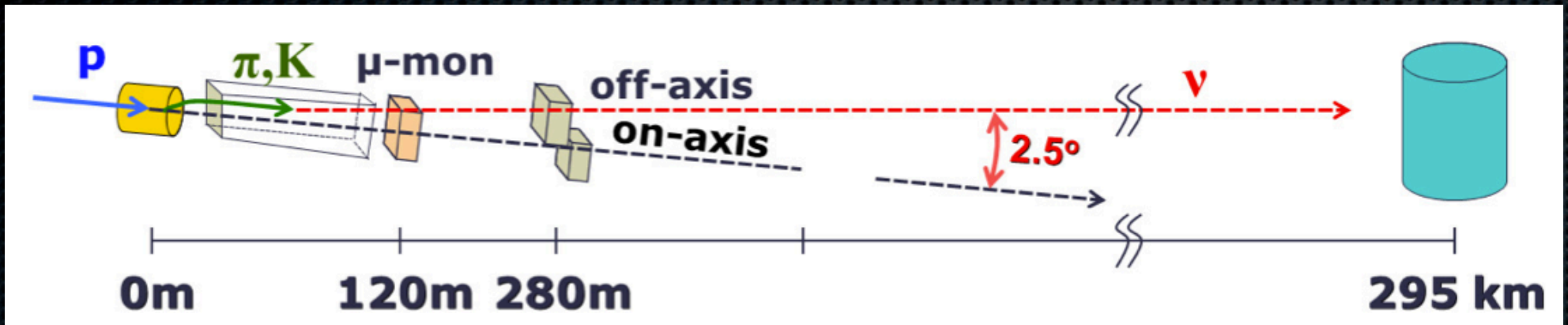


- ✧ At small angles to the beam axis, neutrino energy is insensitive to parent pion energy

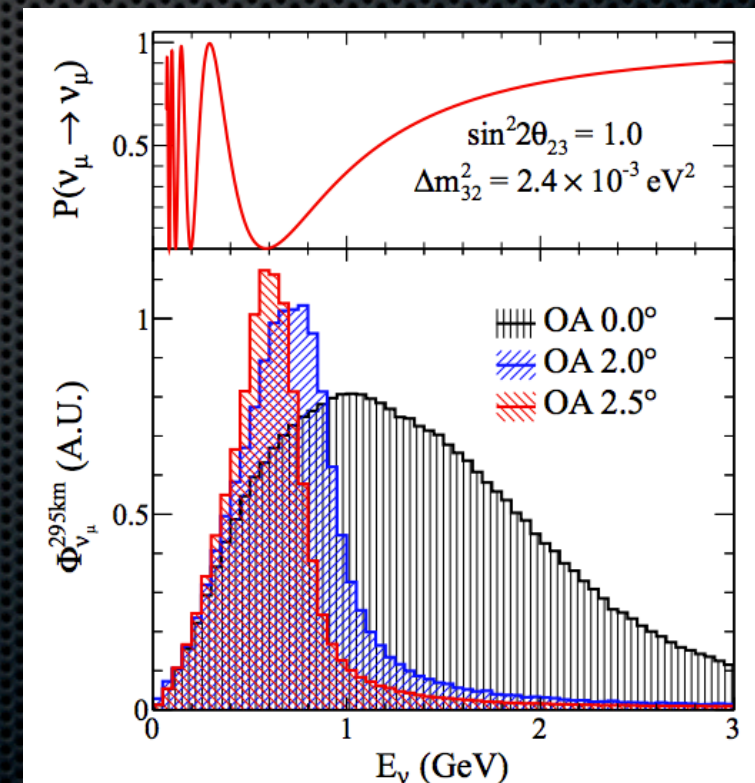




# Off-Axis Beam



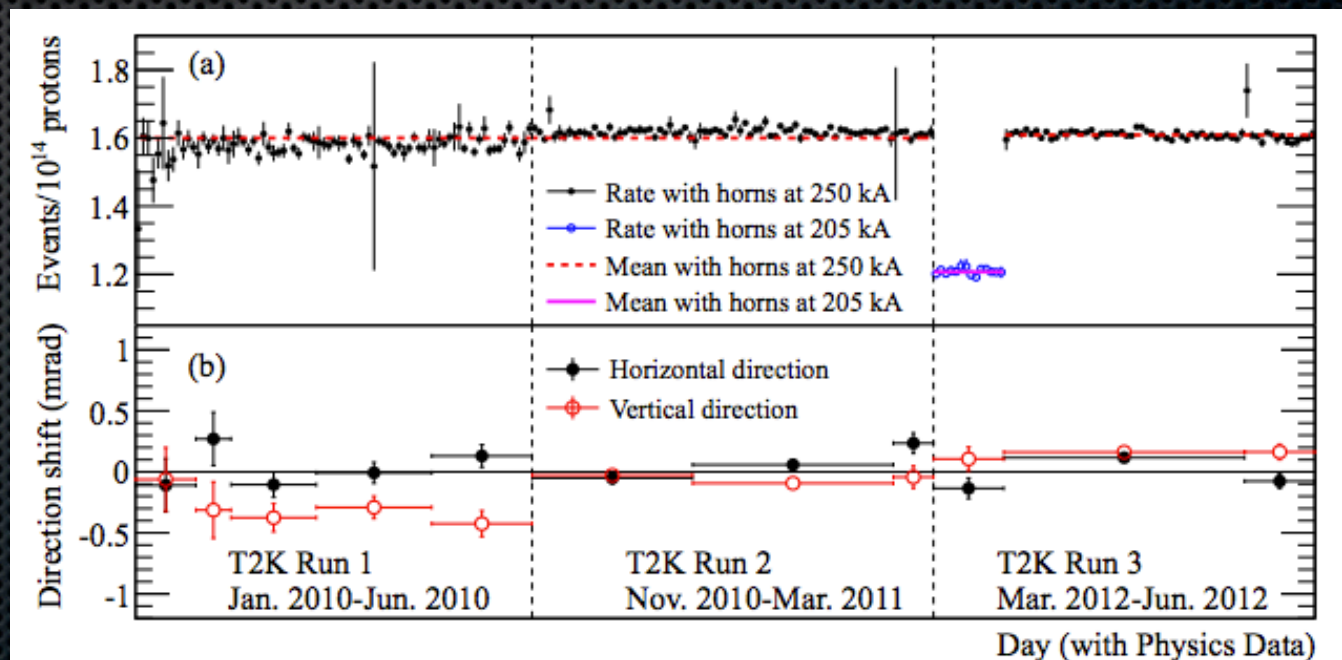
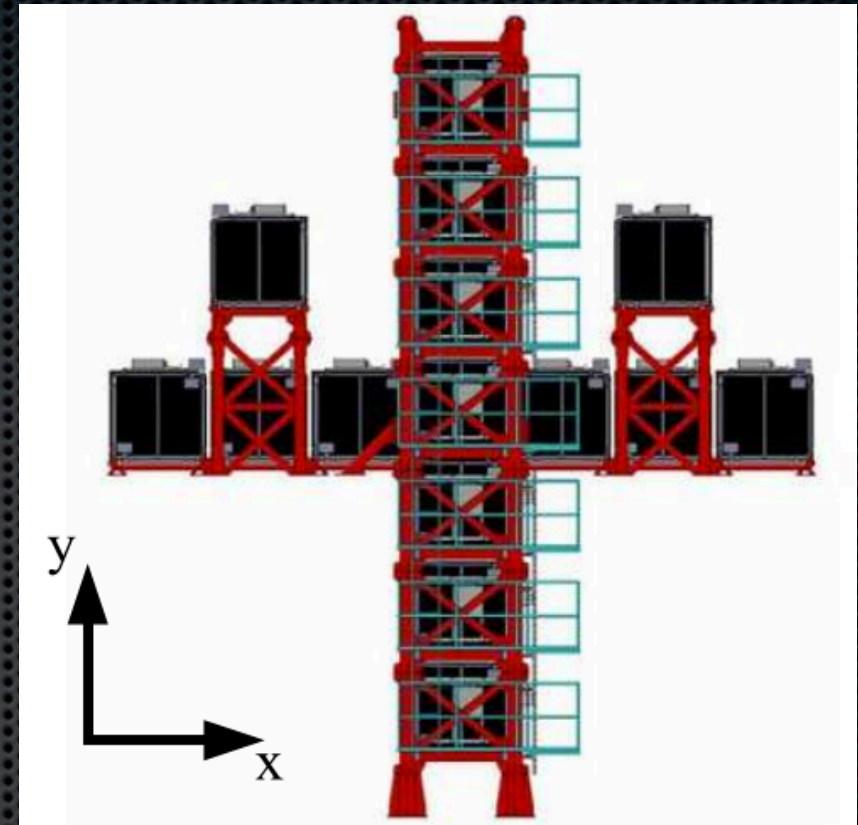
- Peak  $E_\nu$  tuned for oscillation maximum.
- $2.5^\circ$  off axis. Low energy narrow band beam.
- Reduce background from higher energy neutrinos





# On axis INGRID

- ✦ 14 modules consisting of iron and scintillator arranged in a cross pattern
- ✦ Measures profile, direction and intensity of neutrino beam.
- ✦ Rate and beam direction stable over running period.

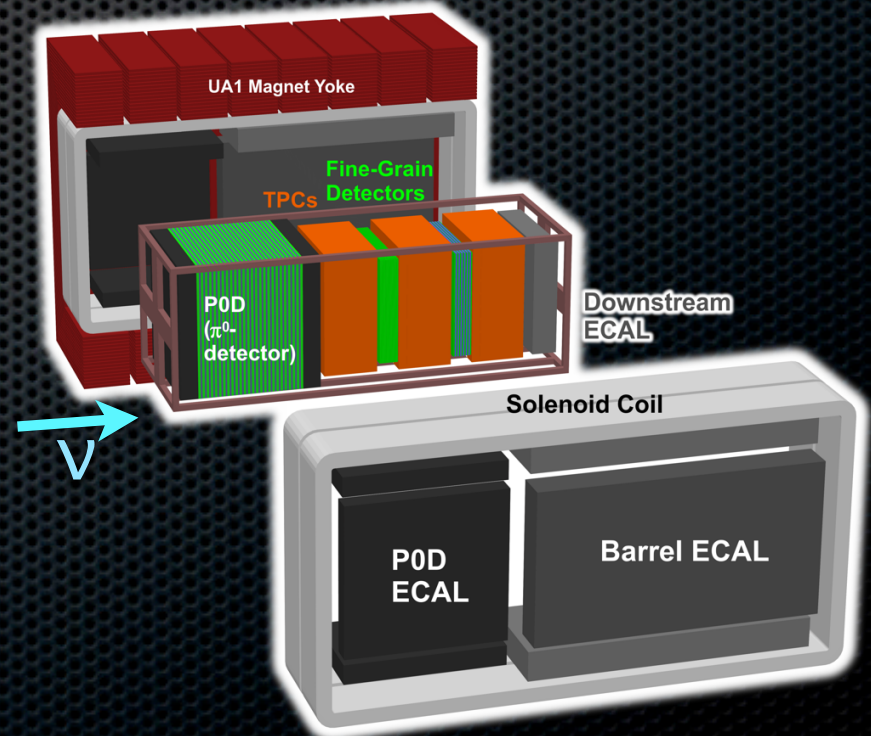
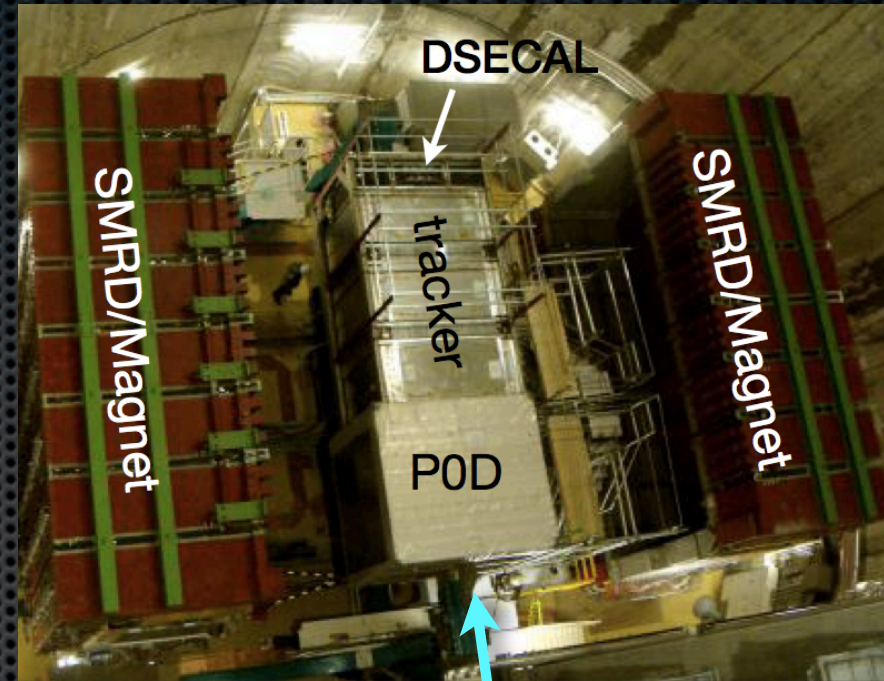


1mrad  $\rightarrow$  2% shift in E



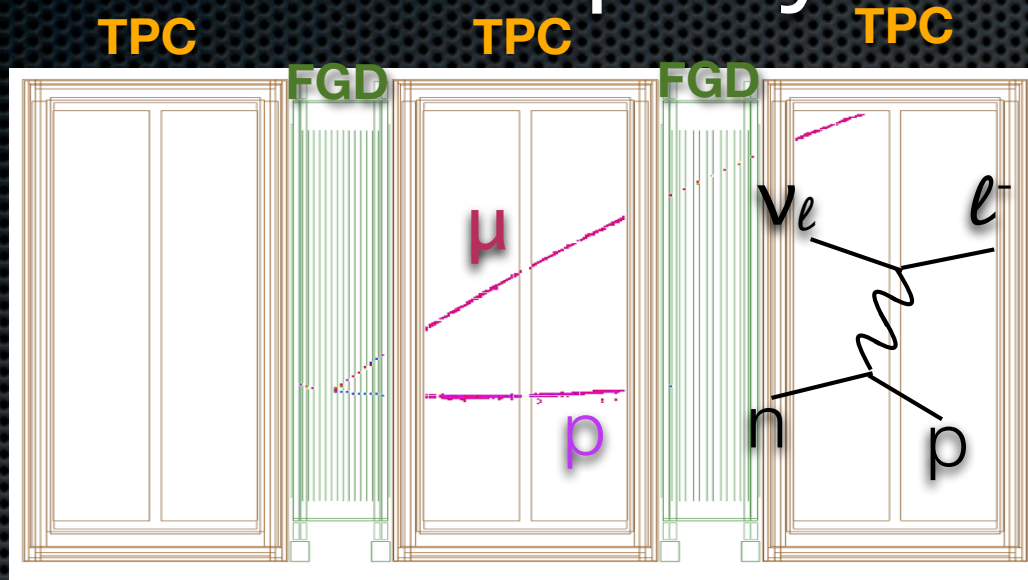
# Off Axis Near Detector

- ND280 (ND=near detector) is located 280 m from production target.
- Multi-Detector complex installed within UA1 magnet.
- Current analyses uses tracker, neutrino interactions in **Fine Grained Detectors** that are measured by **Time Projection Chambers**.
- FGDs provide fiducial mass, particle tracking.
- TPCs measure momenta, particle type.
- Makes measurement of unoscillated beam.  $\nu_\mu$  charged current interactions.
- Crucial in reducing systematic errors for precision oscillation measurements.

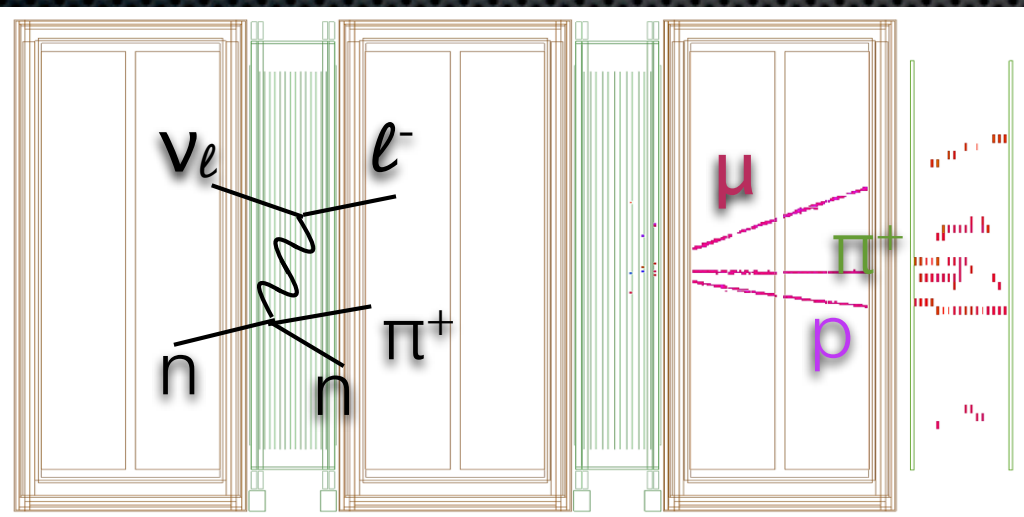




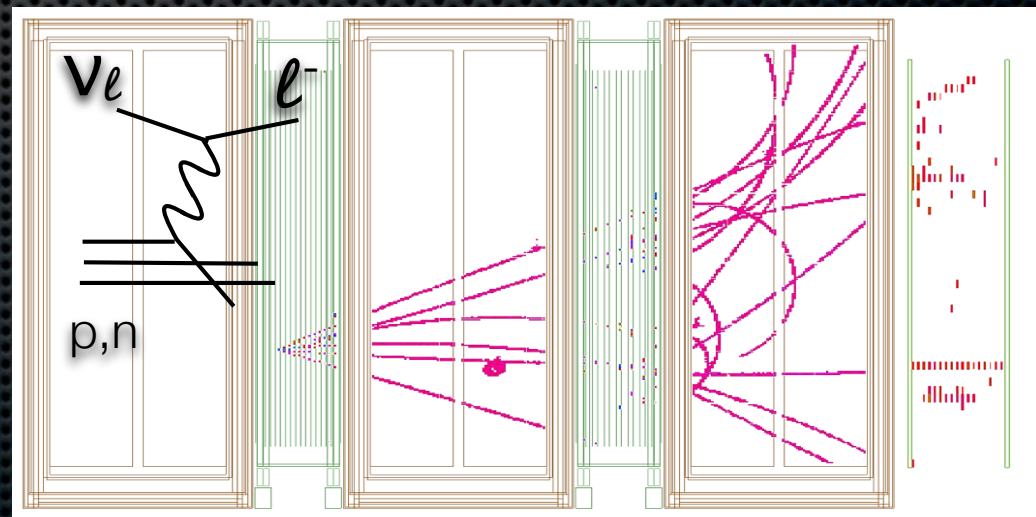
# ND280 Event Display



Quasi Elastic candidate



single pion candidate



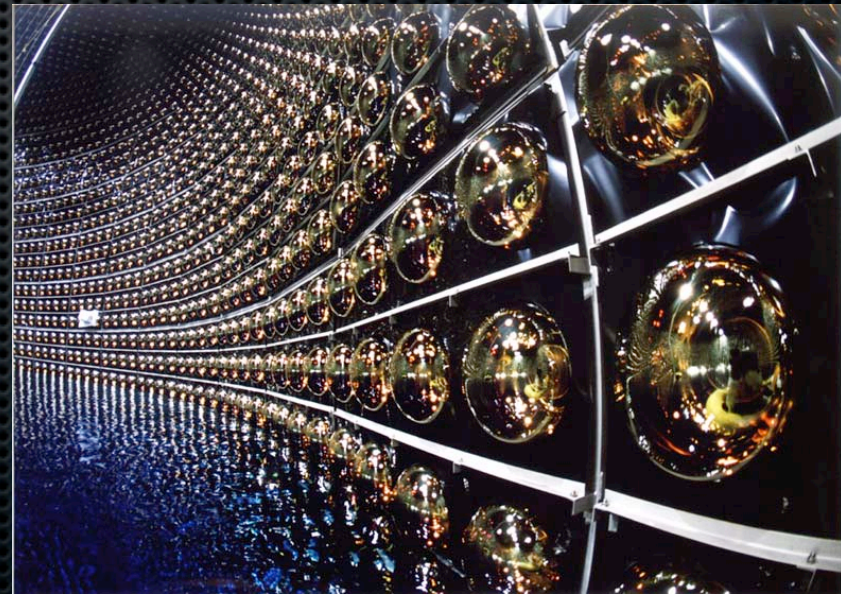
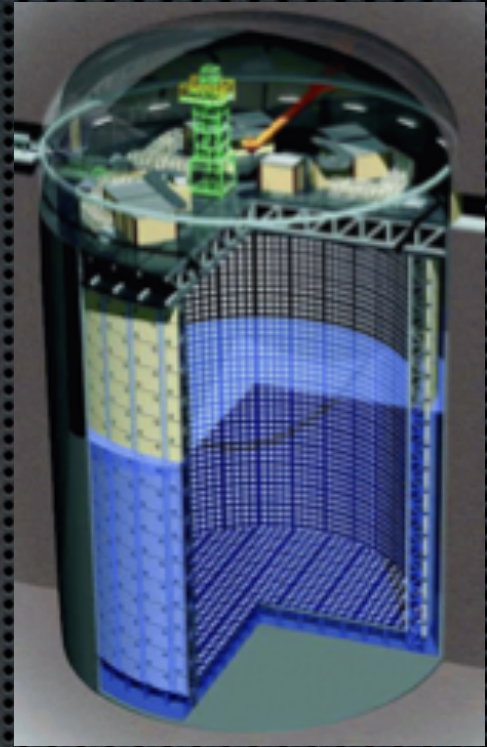
DIS candidate

DIS = deep inelastic scattering. 9



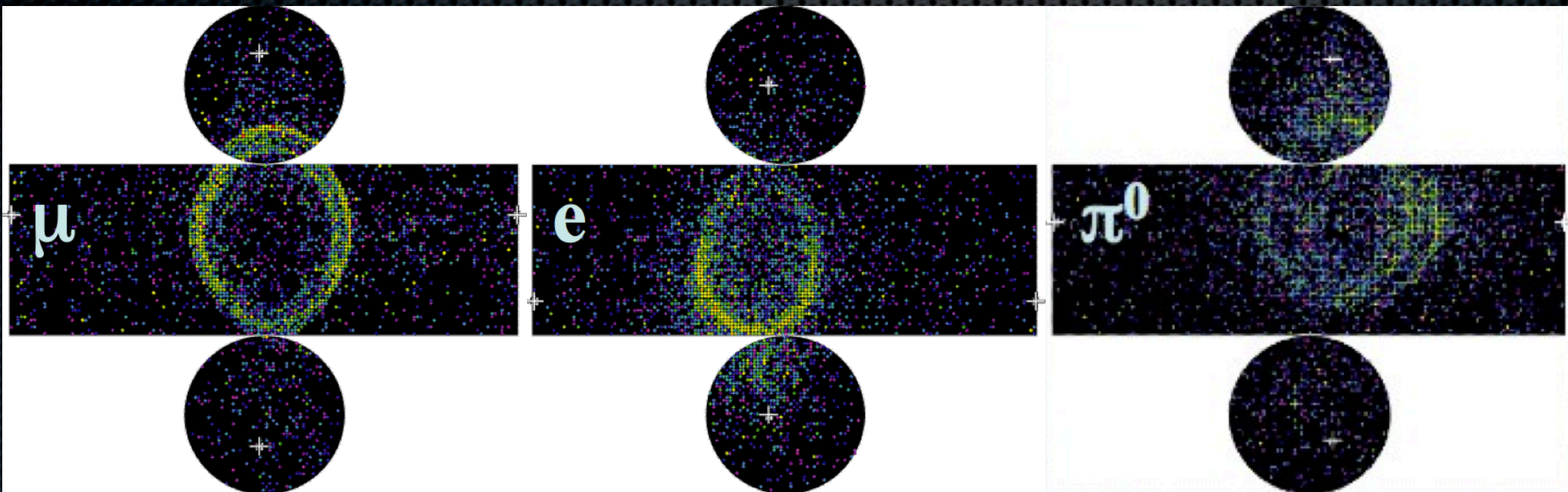
# Super-Kamiokande

- ✦ 50 kton water Cherenkov detector. 22.5 kTon fiducial volume.
- ✦ PMTs line the inner and outer volumes of detector.
- ✦ Charged particles from neutrino interactions produce Cherenkov light. Ring recorded by PMTs.
- ✦ Detector measures direction of recoil particle, momenta, particle type.





# Super-Kamiokande Event Displays



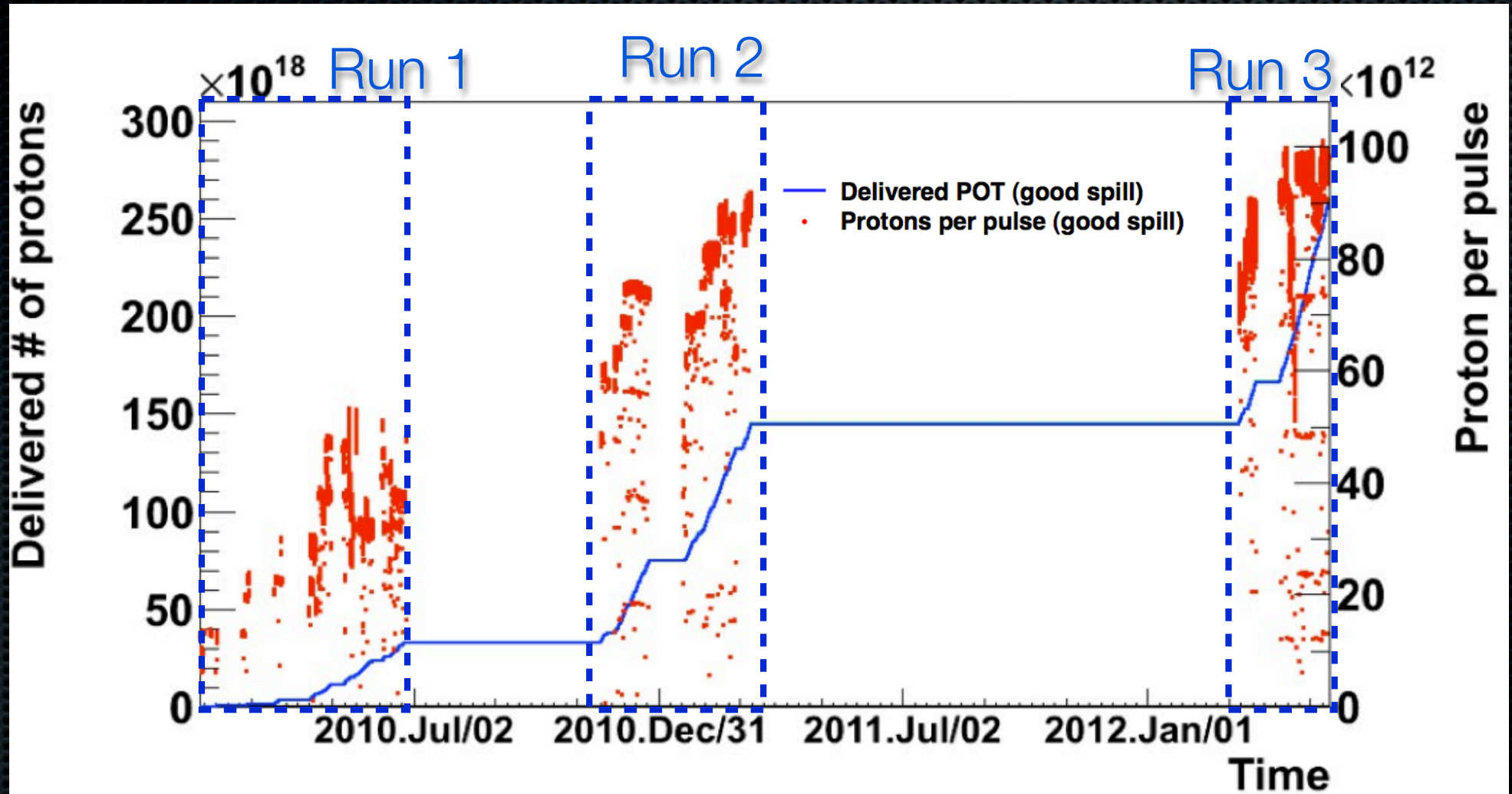
Sharp  $\mu$   
Cherenkov  
ring

Fuzzy e  
Cherenkov  
ring

NC  $\pi^0$  event:  
can mimic e if  
one ring is  
missed.



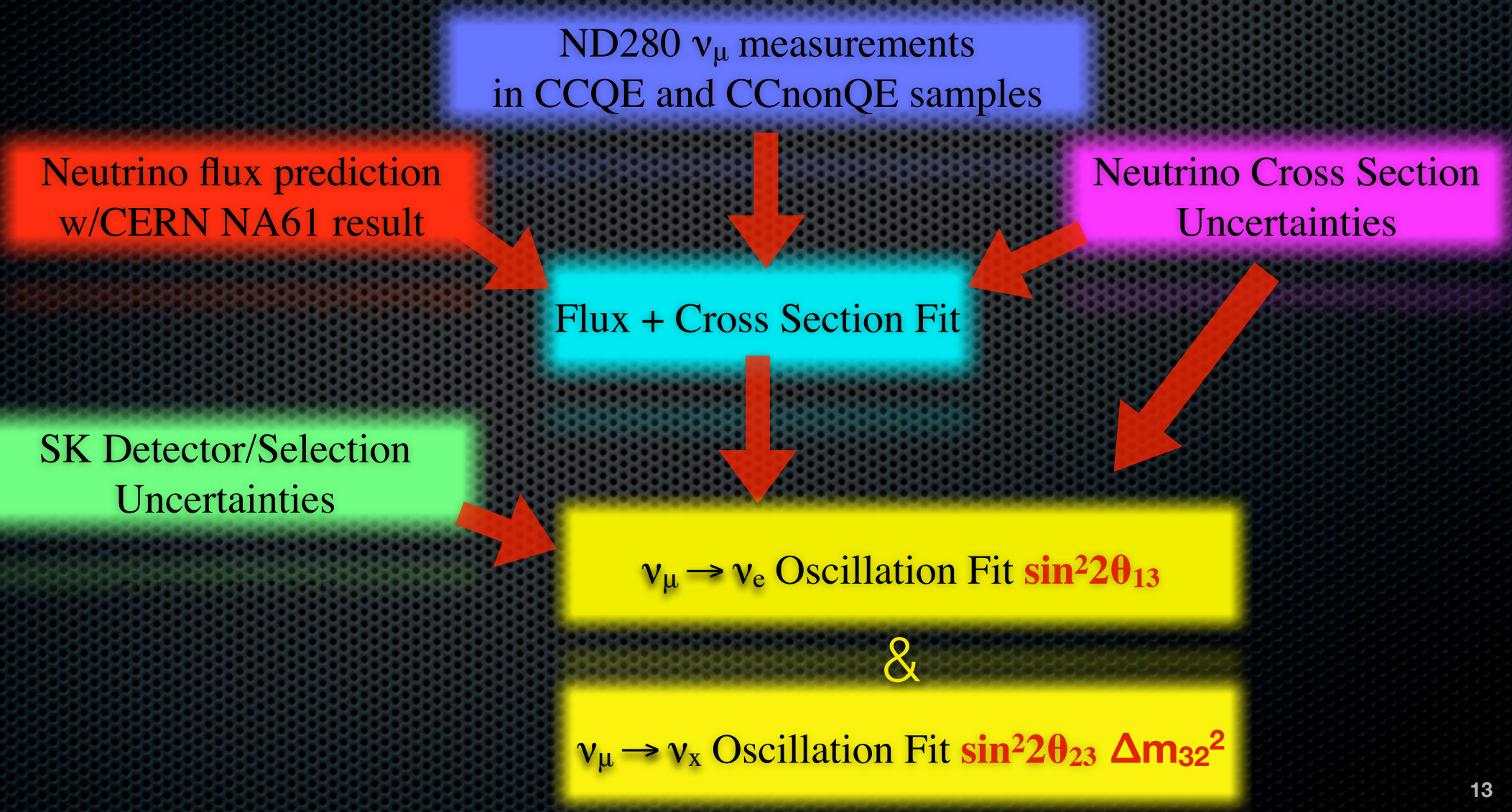
# Analyzed Data



- ✦ Data set runs up to 2012/06/09 (End of Run 3)
- ✦ POT used in this analysis:  $3.01 \times 10^{20}$



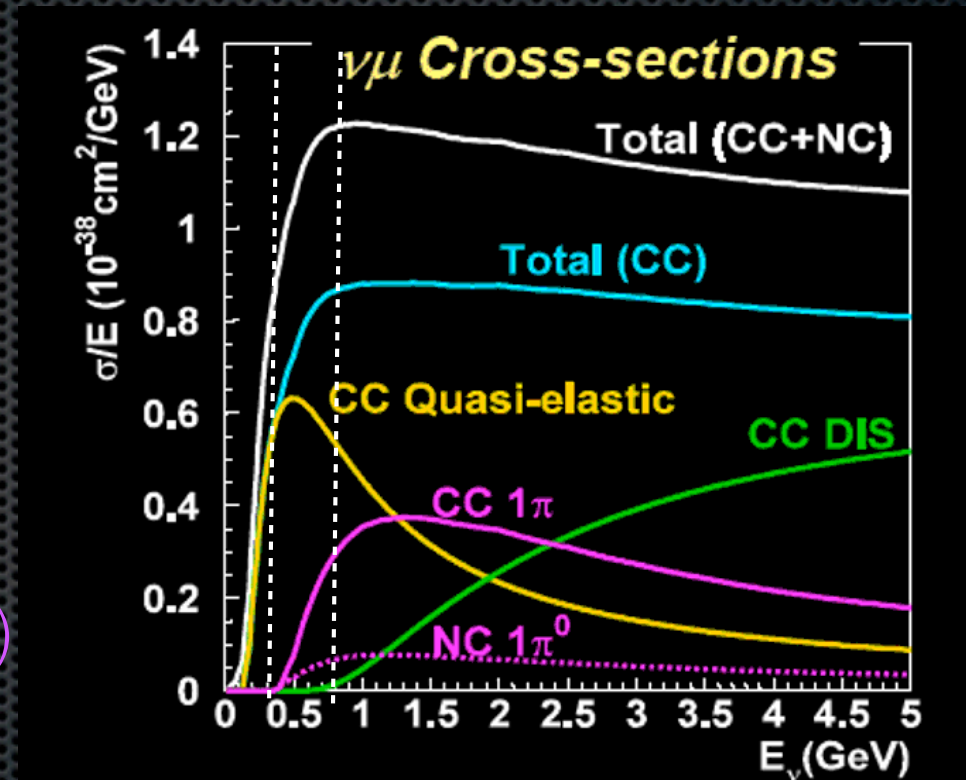
# Oscillation Analyses





# $\nu$ Interactions

- CC (Charged-Current) quasi elastic (CCQE).
- $\nu + n \rightarrow \mu^- + p$  (n in  $^{12}\text{C}$  or  $^{16}\text{O}$ )
- CC (resonance) single  $\pi$  (CC-1 $\pi$ )
- $\nu + n(p) \rightarrow \mu^- + \pi^+ + n(p)$
- DIS (Deep Inelastic Scattering)
- $\nu + q \rightarrow \mu^- + m\pi^{+/-/0} + X$
- CC coherent  $\pi$  ( $\nu + A \rightarrow \mu^- + \pi^+ + A$ )
- NC (Neutral-Current) NC-1 $\pi^0$ , etc...



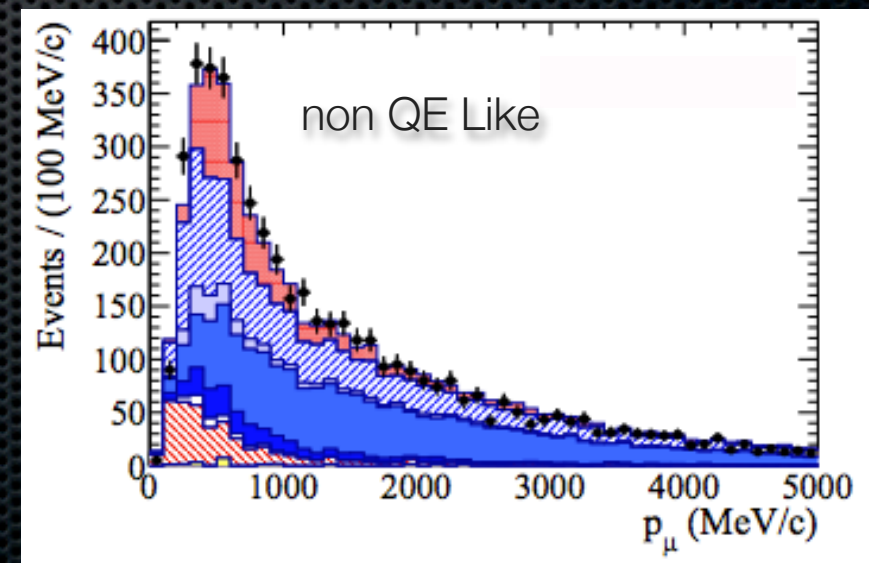
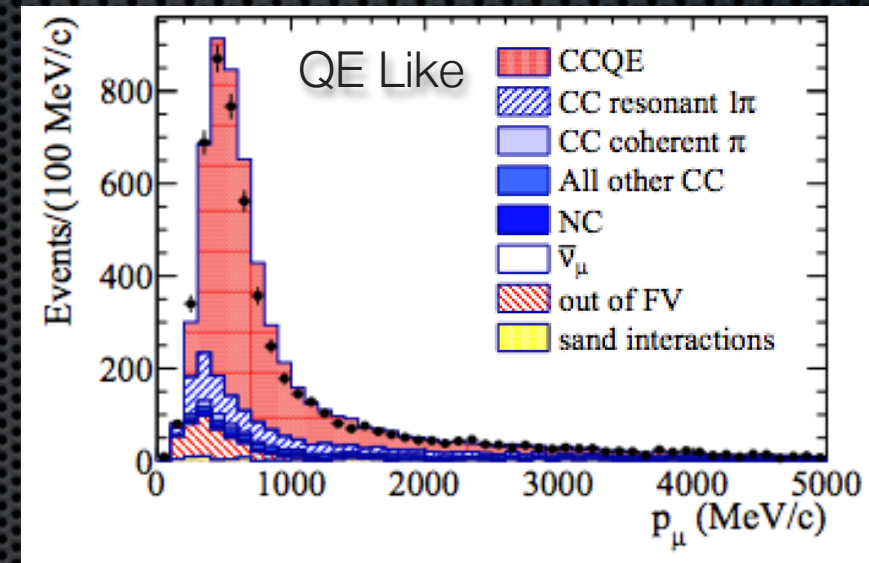
- CCQE Signal Interactions.  
Initial neutrino can be reconstructed from the energy and direction of final lepton

$$E_\nu^{QE} = \frac{m_p^2 - (m_n - E_b)^2 - m_l^2 + 2(m_n - E_b)E_l}{2(m_n - E_b - E_l + p_l \cos(\theta_l))}$$



# ND280 Measurement

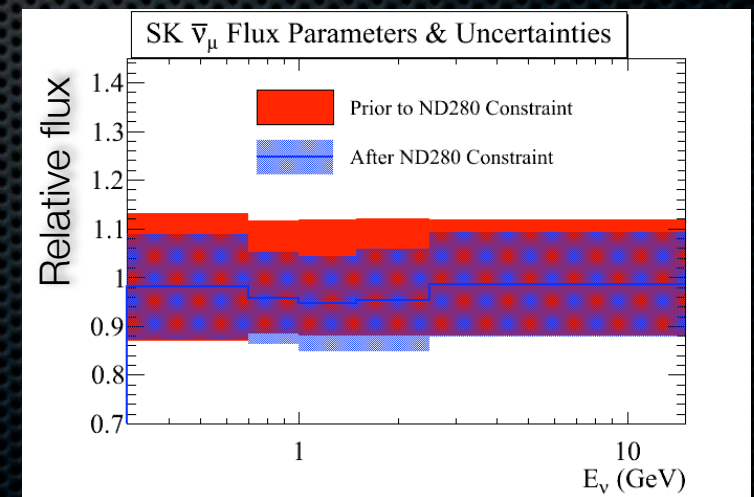
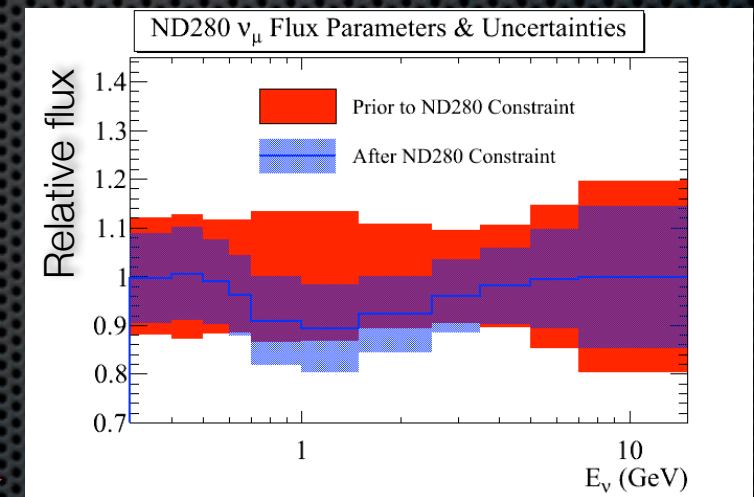
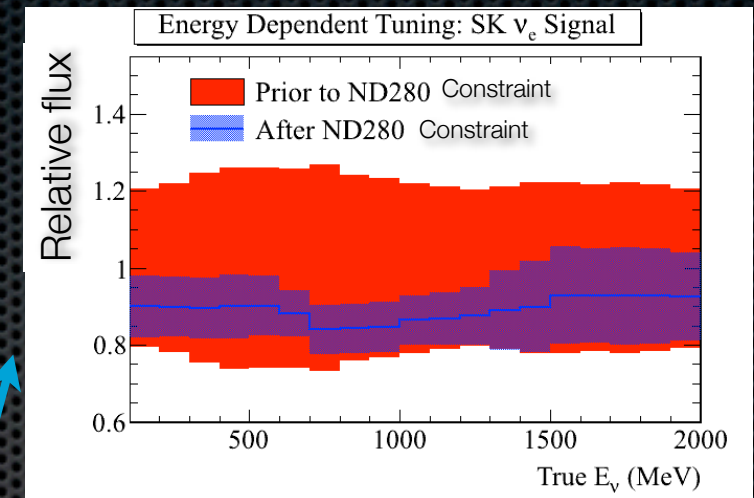
- ✦ Select CC events.
  - ✦ Lepton originating in FGD.
  - ✦ Muon-like  $dE/dx$ , negative curvature in TPC.
- ✦ Divide into QE-like, non-QE-like based on number of tracks.
- ✦ Likelihood fit to CCQE, CCnonQE  $p$ - $\theta$  distributions.
- ✦ Reduce flux and cross section uncertainties





# Flux Constraints

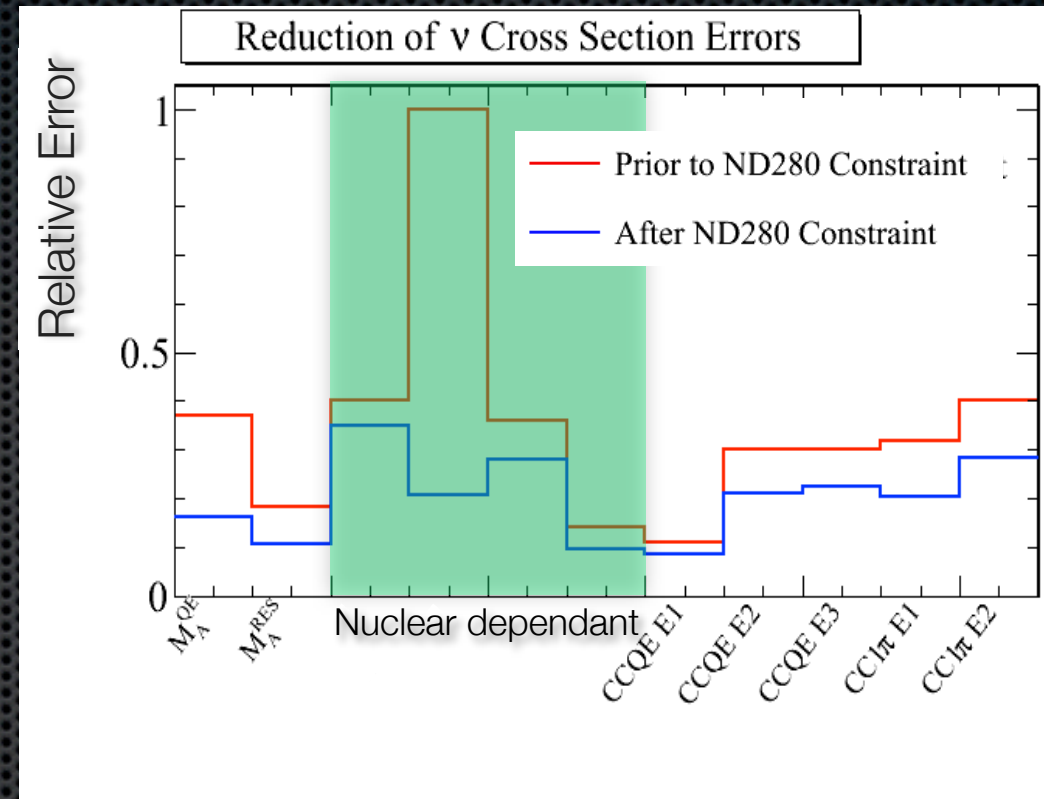
- ✧ Common systematic parameters for ND280 and SK. ND280 used to tune flux and constrain error at SK
- ✧ Fits done with 2 different flux parameterizations.
- ✧  $\nu_e$
- ✧  $\nu_\mu$  and  $\bar{\nu}_\mu$





# Cross Section Constraints

- Parameters with prior uncertainties from Mini-BooNE and other experiments are further constrained at ND280.
- Parameters that do not depend on nuclear target
  - Axial mass for CCQE, CC1 $\pi$
  - Normalization parameters.





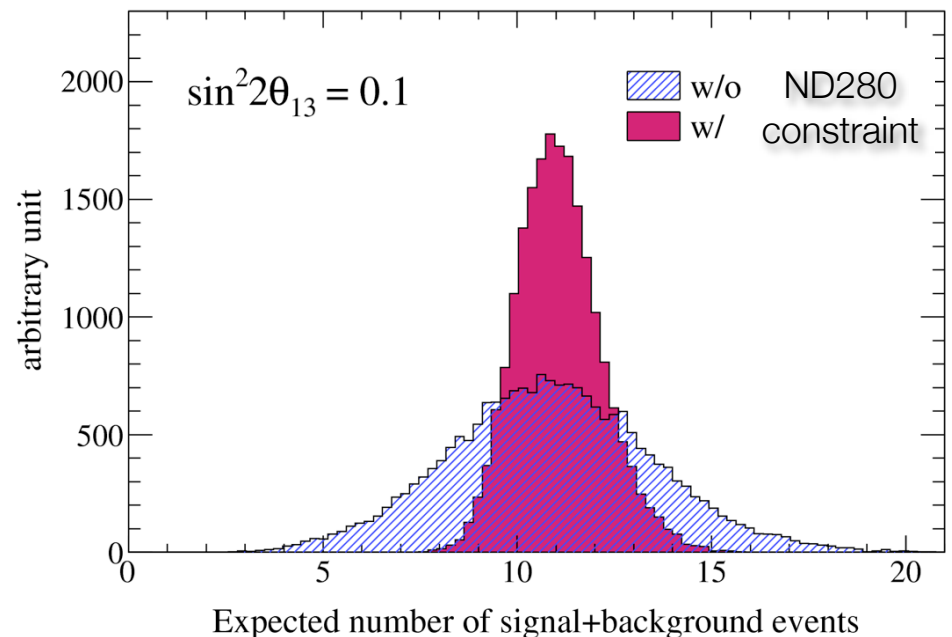
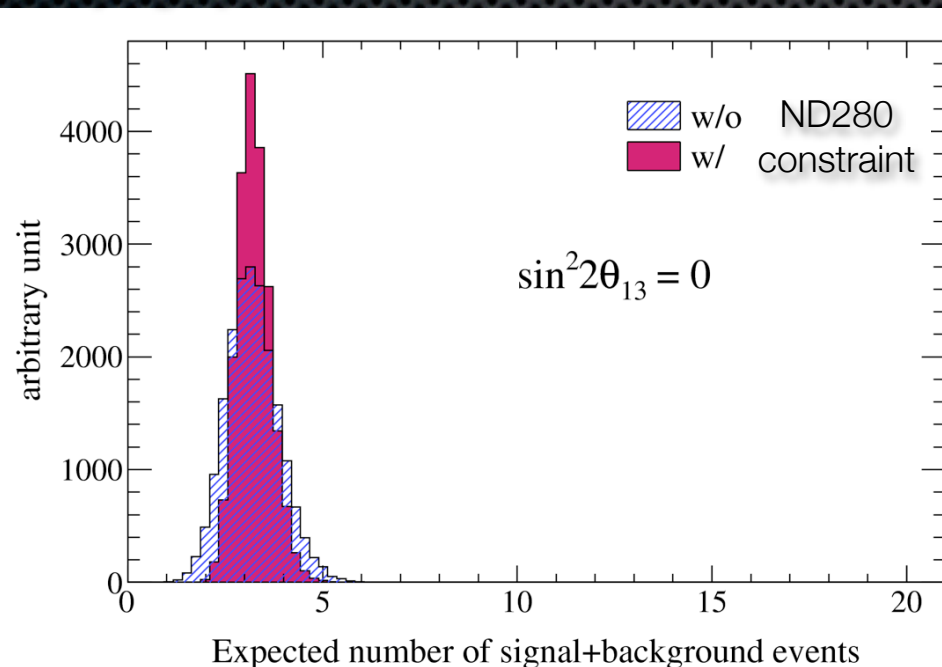
$v_e$  appearance



# Expected number of $\nu_e$ Events

	$\sin^2 2\theta_{13} = 0$	$\sin^2 2\theta_{13} = 0.1$
total	3.3	11.2
CC $\nu_\mu \rightarrow \nu_e$	0.2	8.2
CC $\nu_\mu$	0.06	0.06
CC $\nu_e$	1.8	1.7
NC	1.2	1.2

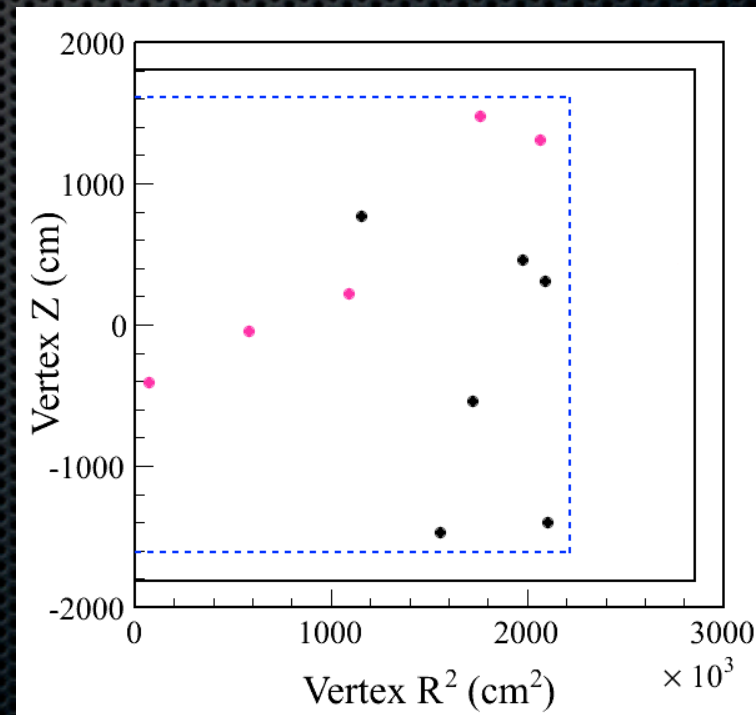
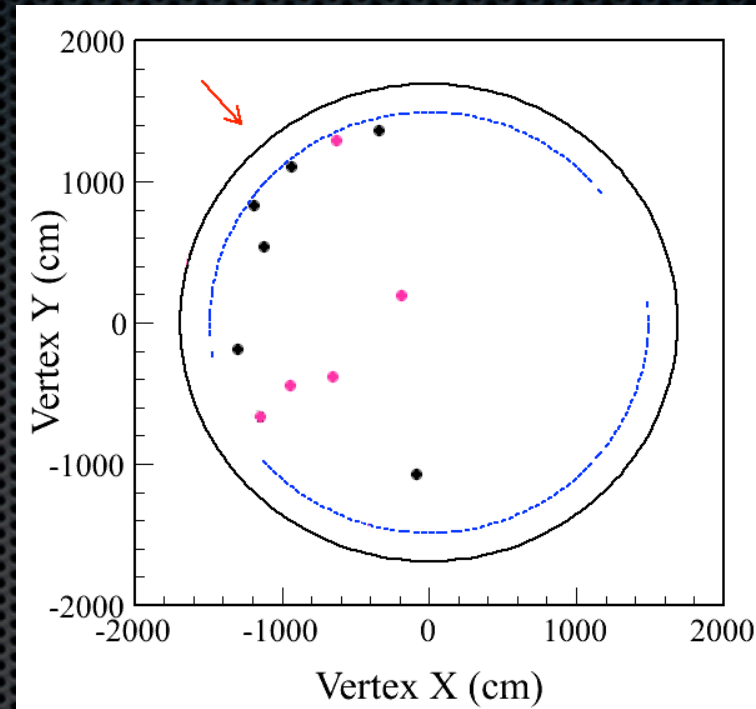
\*varying systematics





# $\nu_e$ candidates

- ✦ 11 candidate events observed.
- ✦ Probability to observe 11 or more events based on the predicted background of  $3.3 \pm 0.4$  (syst.) events is  $9 \times 10^{-4}$ , ( $3.1 \sigma$ )
- ✦ Perform analysis using  $(p, \theta)$  spectrum.



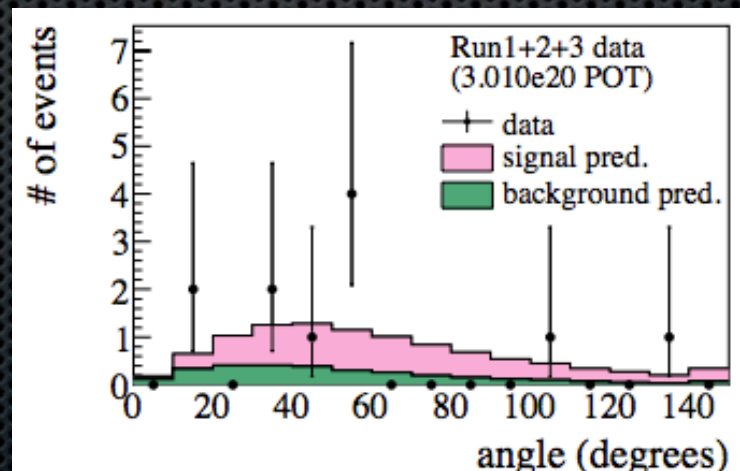
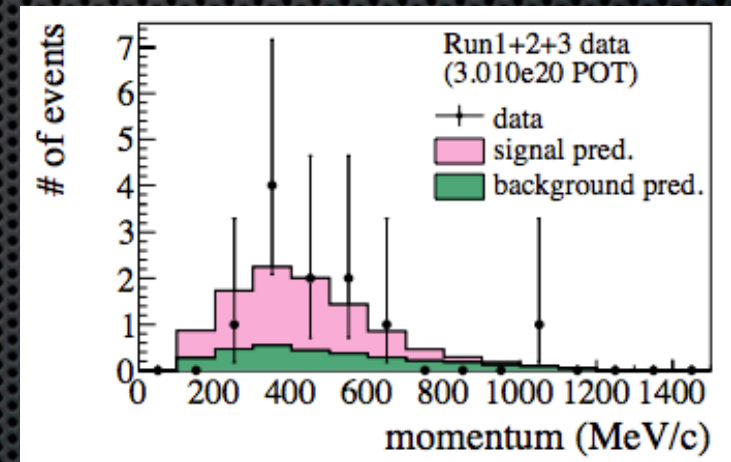


# $\nu_e$ appearance oscillation fit

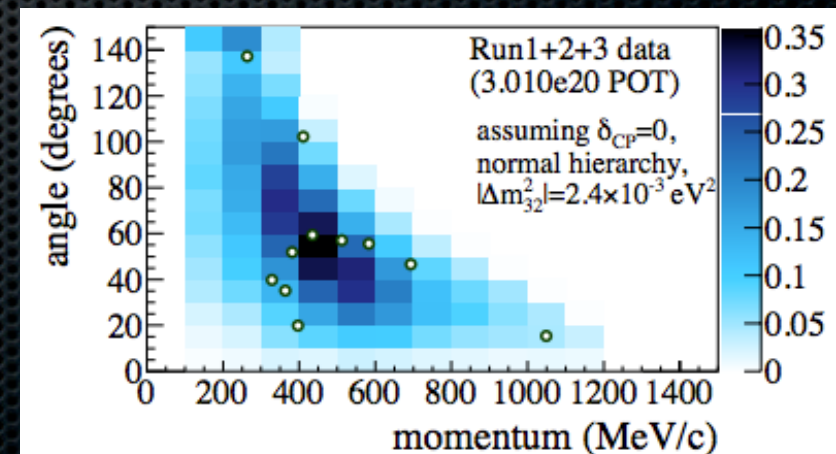
$$\delta_{CP} = 0 \quad \sin^2 2\theta_{23} = 1.0 \quad \Delta m_{32}^2 = 2.4 \times 10^{-3} \text{eV}^2$$

$$\sin^2 2\theta_{13} = 0.088^{+0.049}_{-0.039} \text{ (normal hierarchy)}$$

$$\sin^2 2\theta_{13} = 0.108^{+0.059}_{-0.046} \text{ (inverted hierarchy)}$$



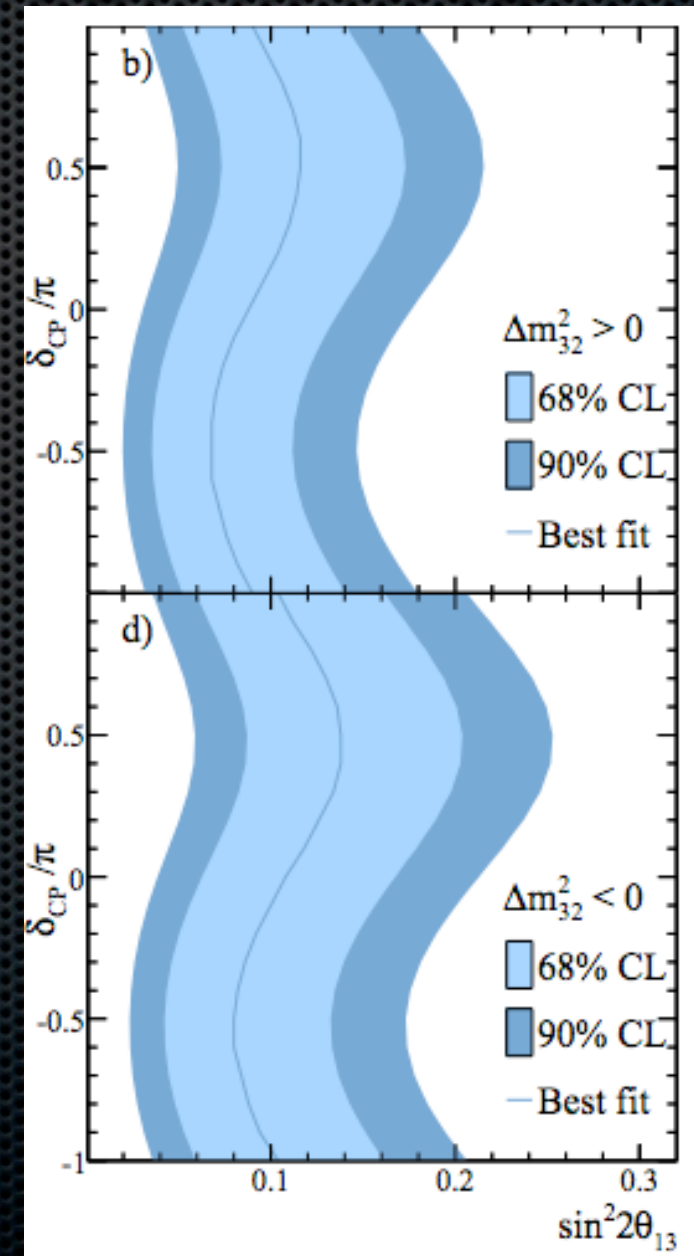
p-value off  $1 \times 10^{-3}$   
 $\theta_{13}=0$  hypothesis





# Oscillation parameter limits

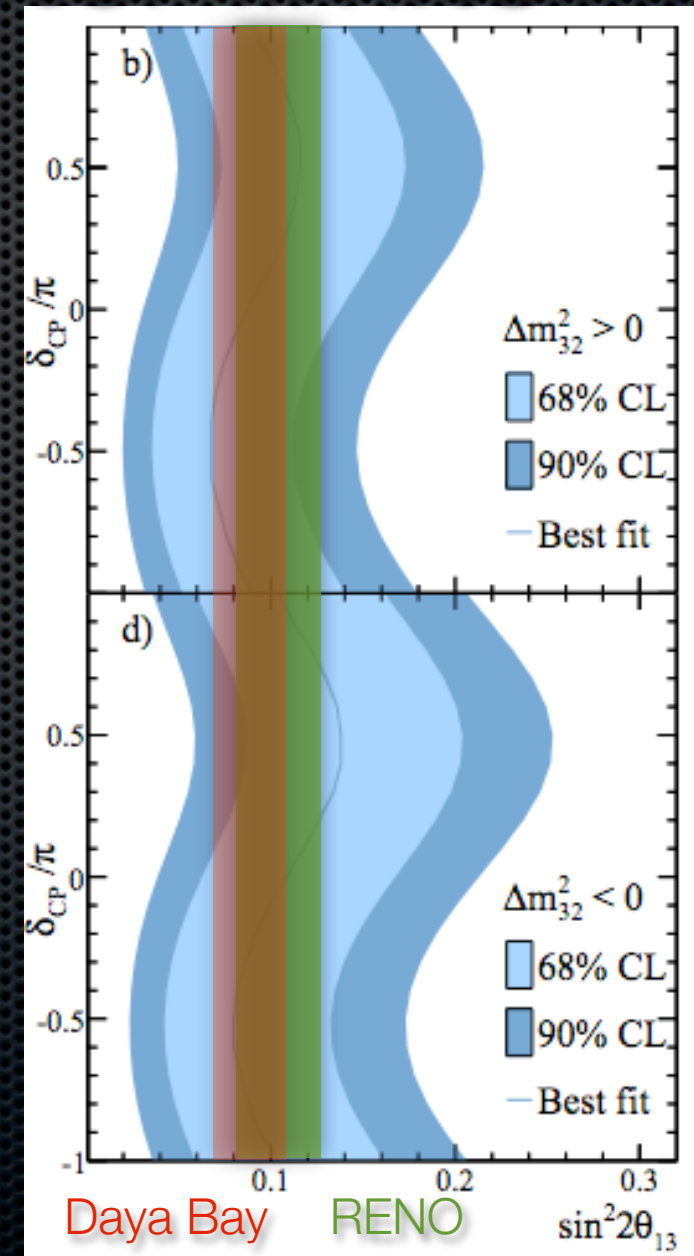
- 68% and 90% confidence intervals scanning over  $\delta$ 
  - Top: Normal hierarchy
  - Bottom: Inverted hierarchy
- Analysis done with 2 other methods
  - Rate + Reconstructed  $E_\nu$
  - Rate only.
- All 3 methods give consistent results.





# Oscillation parameter limits

- ✦ Reactor experimental results (Daya Bay,RENO) consistent with T2K.
- ✦ Reactor experiments non sensitive to  $\delta$  complementary to accelerator experiments.





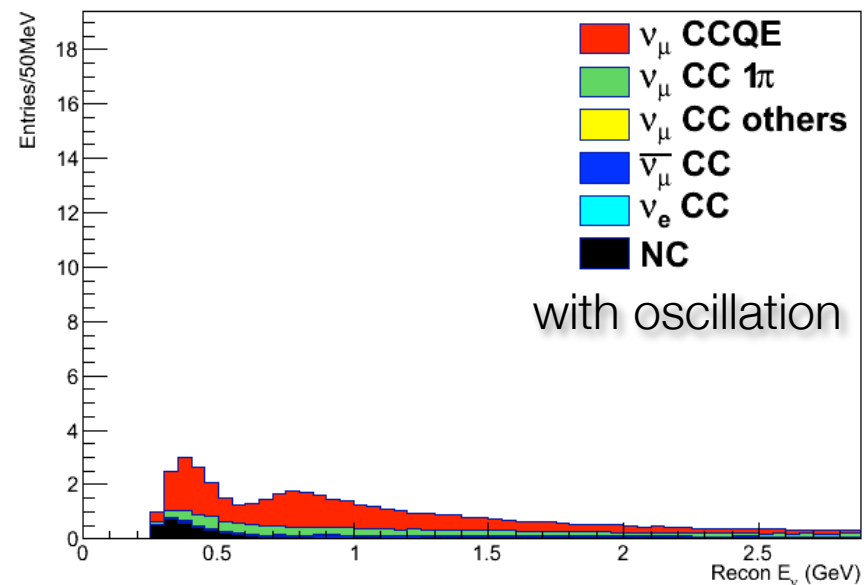
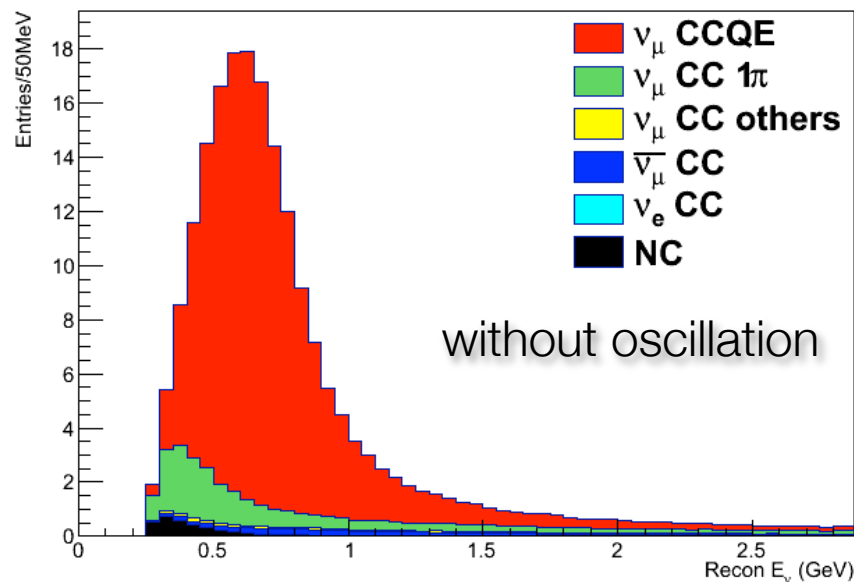
$\nu_\mu$  disappearance



# Expected number of $\nu_\mu$ Events.

Event category	# of pre-calculated events	
	without oscillation	with oscillation
Total	210.46	59.39
CC $\nu_\mu$ signal	200.55	52.17
CC $\bar{\nu}_\mu$ background	6.37	3.56
CC $\nu_e$ background	0.03	0.03
CC $\bar{\nu}_e$ background	0.00	0.00
Appearance $\nu_e$ background	0.00	0.12
NC background	3.51	3.51

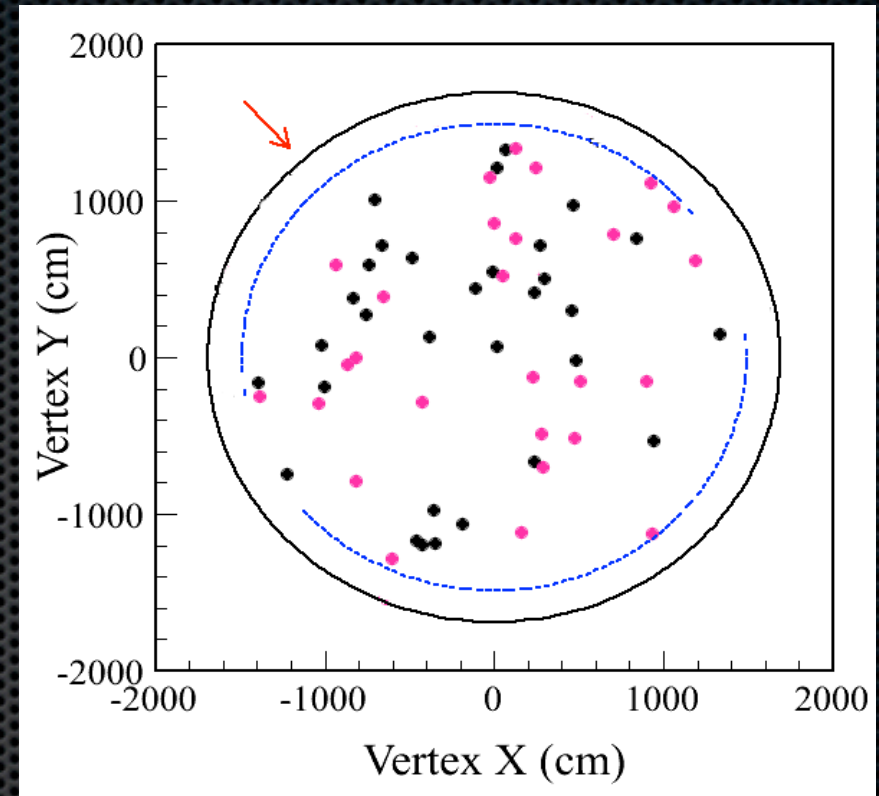
	w/o ND280 constraint	w ND280 constraint
Total Error(%)	24.7	8.3
without oscillation		





# $\nu_\mu$ candidates

- ✦ 58 candidate events observed
- ✦ Likelihood ratio fit binned in reconstructed energy. Compare the observed to expected in 73 variable width bins, concentrated in oscillation region.



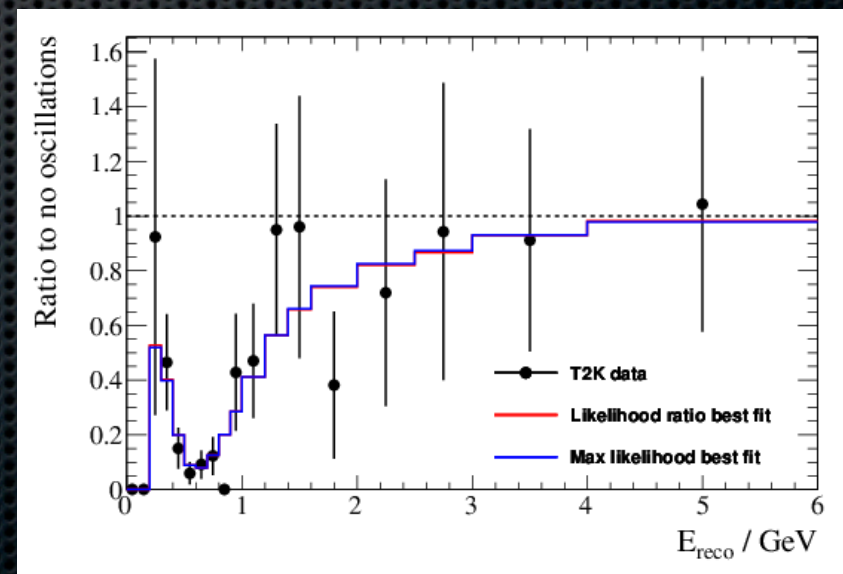
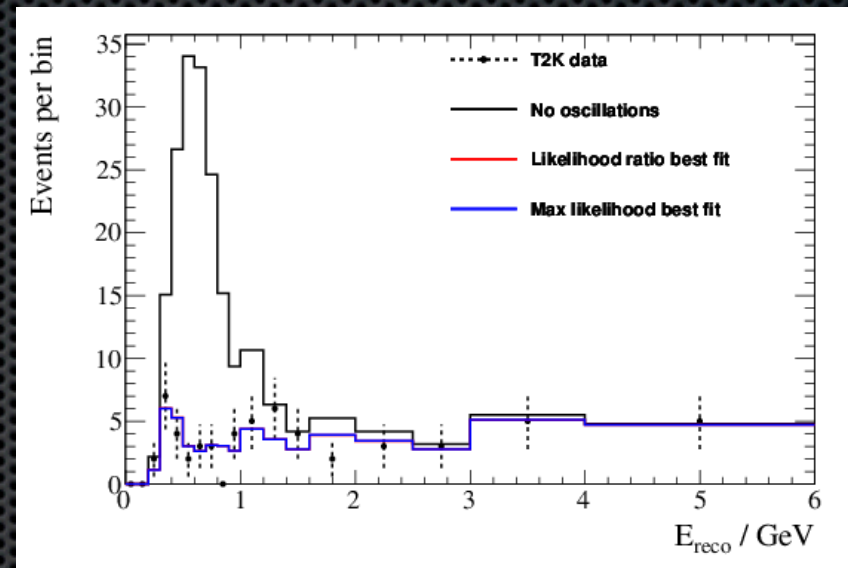
- ✦ Oscillation probability calculated using 3 neutrino flavours,  $\theta_{13}$  using average of reactor experiments
- ✦ Using normal hierarchy, earth density  $2.6 \text{ g/cm}^3$ ,  $\delta = 0$

$\sin^2 2\theta_{13}$	0.098
$\sin^2 2\theta_{12}$	0.857
$\Delta m_{21}^2 (\text{eV}^2)$	$7.5 \times 10^{-5}$



# $\nu_\mu$ disappearance oscillation fit

- ✧ Deficit at survival probability minimum.
- ✧ Analysis done with likelihood ratio fit and Alternative method using unbinned maximum likelihood consistent





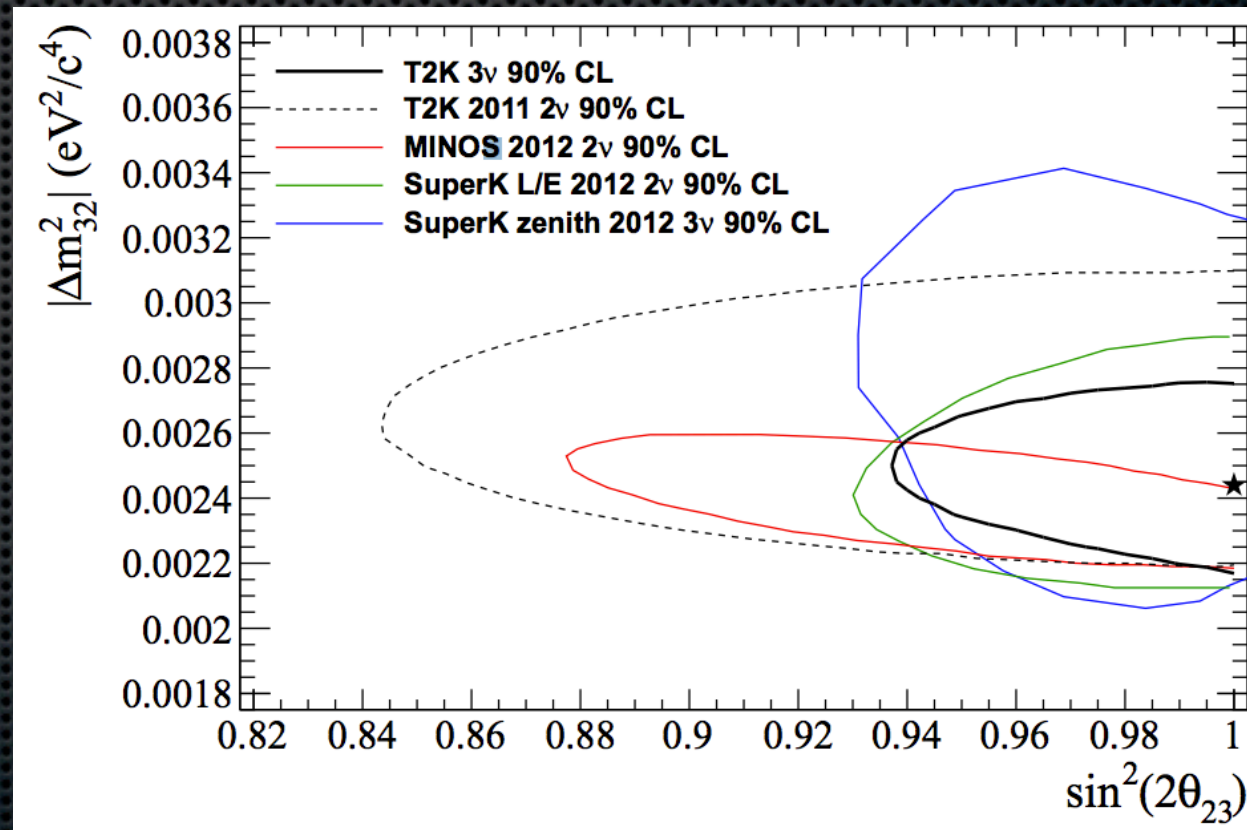
# Oscillation parameter limits

90% CL

$$2.14 \times 10^{-3} \text{eV}^2 < |\Delta m_{32}^2| < 2.76 \times 10^{-3} \text{eV}^2$$

$$\sin^2 2\theta_{23} > 0.957$$

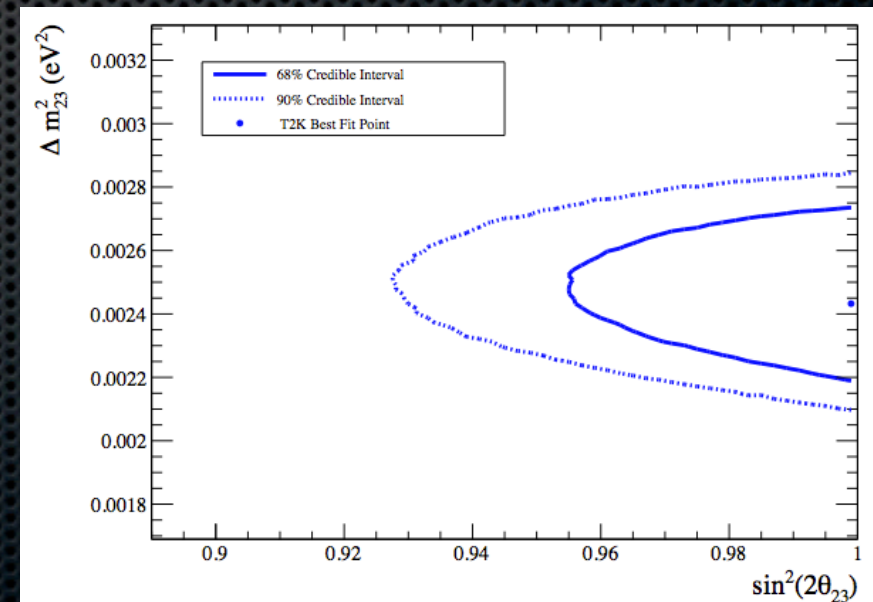
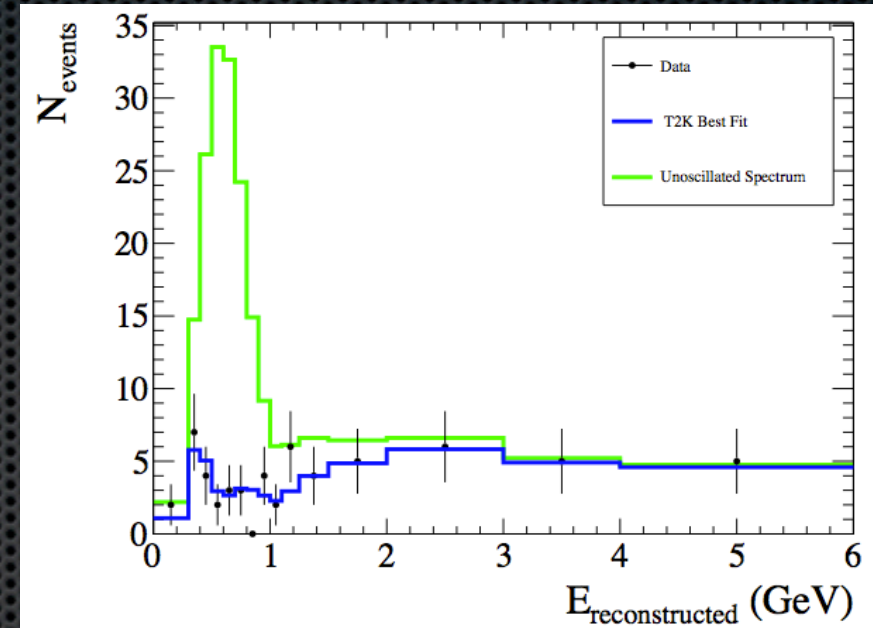
- ✦ Measures maximal mixing.
- ✦ Statistical error dominant





# Simultaneous Fit

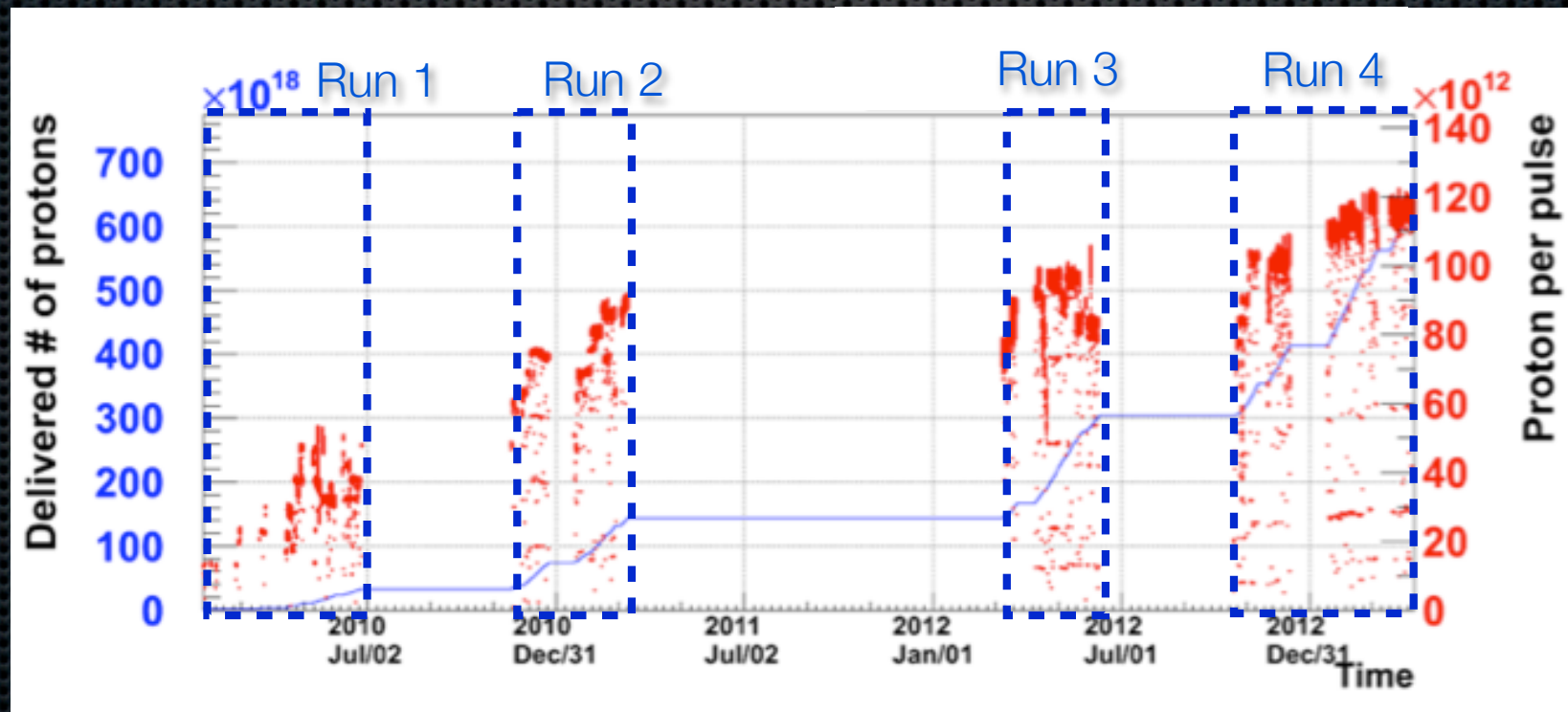
- ✦ Complementary study was done doing a simultaneous fit of ND280 and SK.
- ✦ Monte Carlo Markov Chain (MCMC) to find posterior distribution
- ✦ MCMC analysis produced Bayesian credible intervals.
- ✦ Credible/confidence intervals similar shape and size.





# Future Prospects

- ✦  $3.01 \times 10^{20}$  POT is 4% of T2K goal, measurements are still statistics limited.
- ✦ Data collection ongoing  $6.39 \times 10^{20}$  POT as of April 12th





# Future Prospects

- ✦ Analysis upgrades
  - ✦ Upgraded SK fitting algorithm. Upgraded SK cuts, error evaluation. New samples used at ND280.
- ✦ Move to simultaneous fitting of  $\nu_e$  &  $\nu_\mu$  samples
- ✦ Task force formed to study future sensitivity of T2K, best combination of  $\nu \bar{\nu}$  running. Combining T2K with other experiments to search for the  $\theta_{23}$  octant, mass hierarchy and CP violation.



# T2K Collaboration

~500 collaborators from 56  
institutions, 11 nations





Back up



# $\nu$ Oscillations

- ✦ The flavour state of the neutrino,  $\nu_\alpha$  can be expressed as a superposition of mass states  $\nu_i$ .

$$|\nu_\alpha\rangle = \sum U_{\alpha i} |\nu_i\rangle$$

- ✦ Three neutrino flavours, neutrino mixings are described by the 3x3 PMNS matrix.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$



# $\nu$ Oscillations

- PMNS matrix often parameterized as

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij}$$

- Measured with atmospheric and long baseline  $\nu$ .  $\theta_{23} \approx \pi/4$
- Measured with solar, reactor  $\nu$ .  $\theta_{12} \approx \pi/6$
- Measured with reactor, long baseline  $\nu$ .  $\theta_{13} \approx \pi/20$
- Very different than the CKM matrix!
- CP violating phase  $\delta$  has not yet been measured.



# CP Violation in Lepton Sector

- ✦ CPV in quarks (CKM matrix) does not explain the matter antimatter asymmetry in the Universe.
- ✦ What about the leptons?
- ✦ CPV in neutrinos could give hints towards matter antimatter asymmetry.
- ✦ The CP asymmetry of neutrinos in terms of  $\nu_e$  appearance.

$$\frac{P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} \simeq \frac{\Delta m_{21}^2 \sin 2\theta_{12}}{4E_\nu \sin \theta_{13}} \sin \delta.$$

- ✦ Measurements of  $\theta_{13}$  made in the past few years.



# $\theta_{13}$ at T2K

- ✦ T2K measures  $\theta_{13}$  via  $\nu_e$  appearing in a  $\nu_\mu$  beam.
- ✦ Appearance dependant  $\theta_{13}$  as well as CPV term, mass hierarchy,  $\theta_{23}$  octant .

$$P(\nu_\mu \rightarrow \nu_e) \sim \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right) +$$

(CPV term) + (matter term)

- ✦ Up to eight-fold ambiguity in determining  $\theta_{13}$  and  $\delta$  from  $P(\nu_\mu \rightarrow \nu_e)$

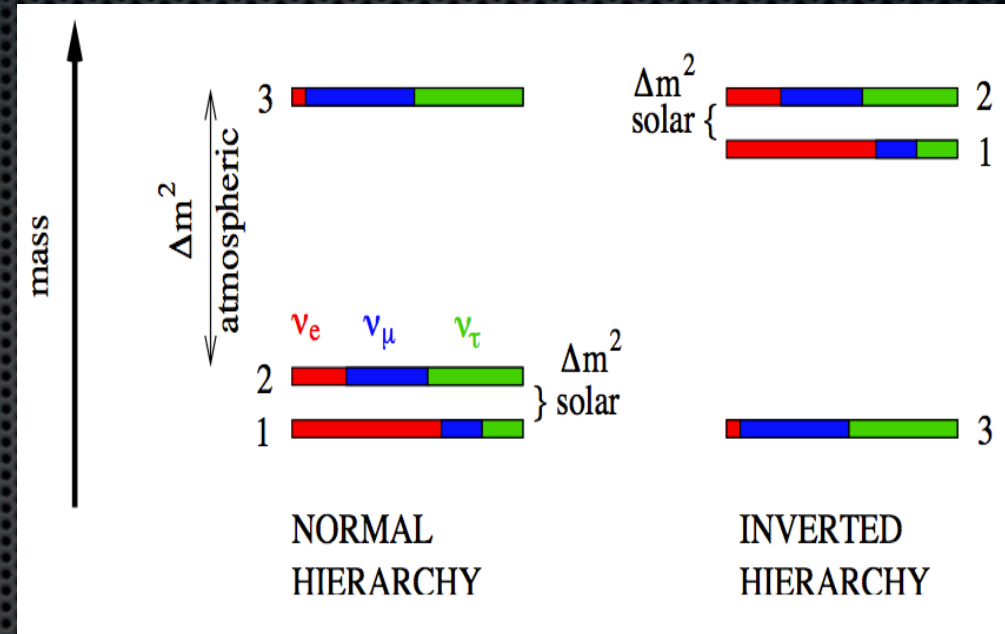


# Unknowns

- ✧ Mass hierarchy still unknown.

- ✧  $\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$

- ✧  $\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}^2$

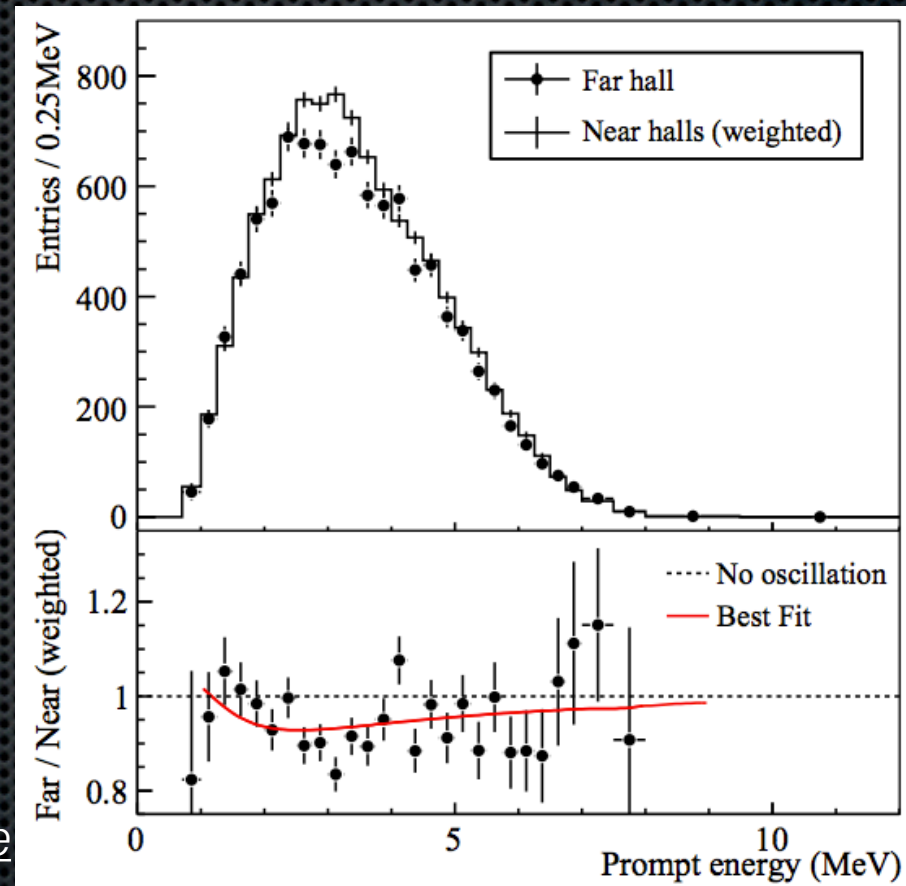


- ✧  $\theta_{23}$  is still consistent with maximal mixing  $\pi/4$
- ✧ Deviation of  $\theta_{23}$  from maximal mixing? Lower or higher octant?



# The past few years.

- June 2011: T2K. Electron neutrino appearance
- $\sin^2 2\theta_{13} = 0.11 \pm 0.044$  ( $2.5 \sigma$ ) (at  $\delta_{cp} = 0$ , NH).
  - PRL 107, 041801
- March 2012: Daya Bay. Electron neutrino disappearance
  - $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{sys})$  ( $5.2 \sigma$ )
  - PRL 108, 171803
- April 2012: RENO. Electron neutrino disappearance
  - $\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{sys})$  ( $4.9 \sigma$ )
  - arXiv:1204.0626

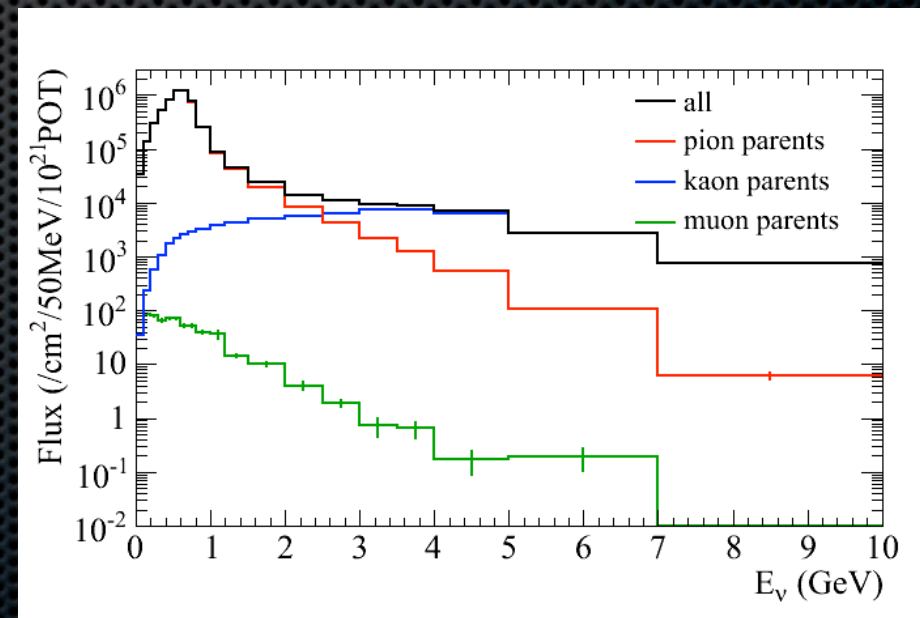
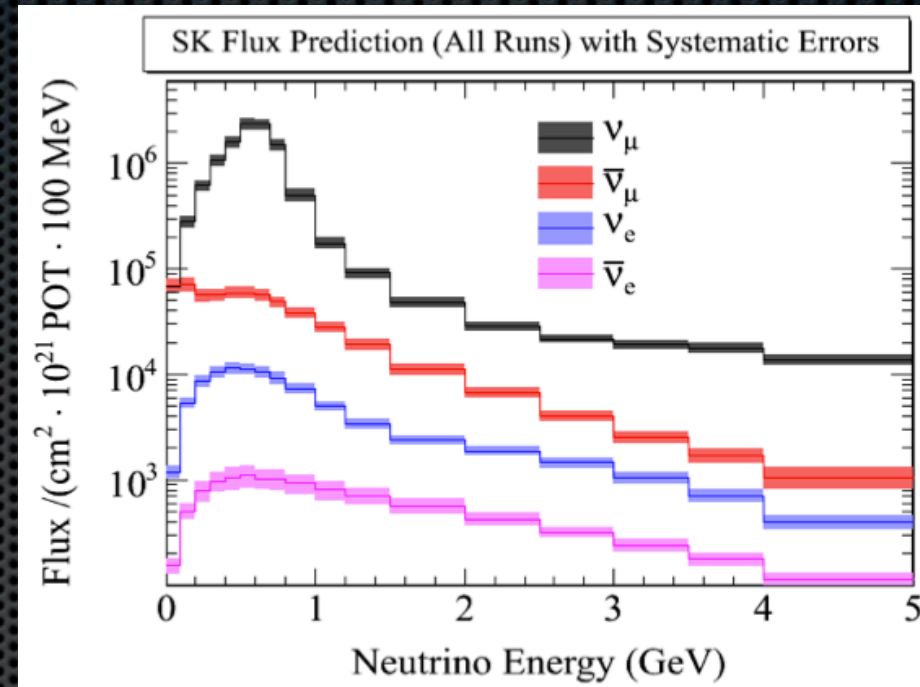


Daya Bay reactor  
neutrino disappearance



## Neutrino flux prediction w/CERN NA61 result

- ✦ Uncertainty in flux found from proton beam profile, hadron production uncertainties.
- ✦ Kaon, pion production measured from NA61 experiment with same target material, beam energy as T2K.
- ✦ Tuned FLUKA + GEANT3 simulation used to estimate fluxes at ND280 and SK
- ✦ Beam flux uncertainty at Super Kamiokande  $\sim 15\%$  before ND280 constraint.

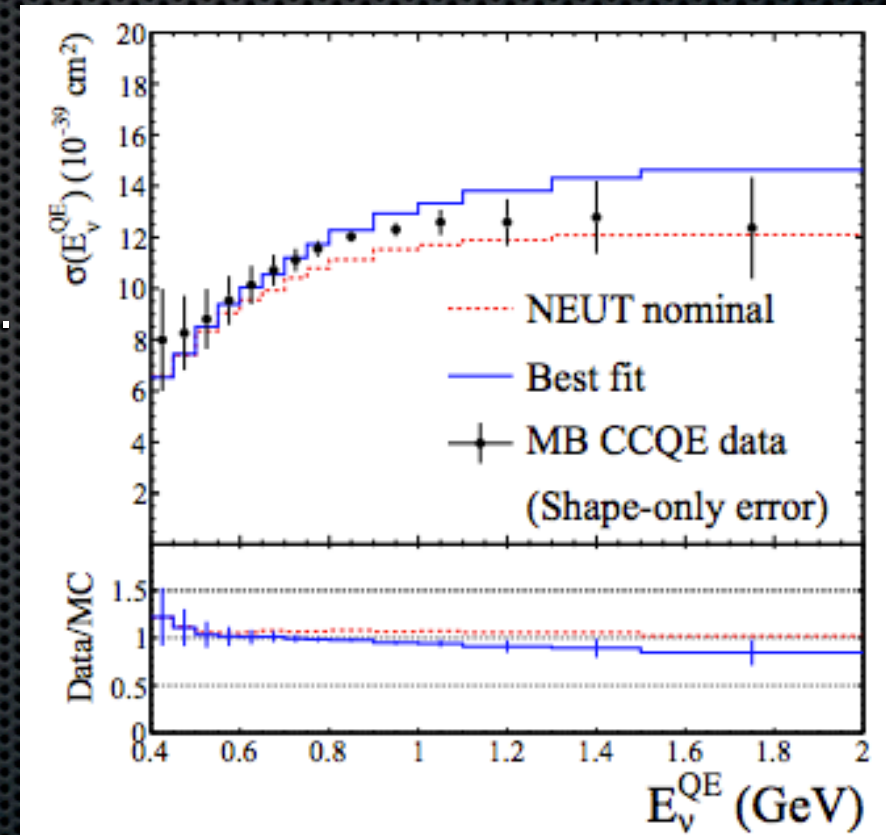


$\nu_\mu$  flux broken down by parent that produces  $\nu$



## Neutrino Cross Section Uncertainties

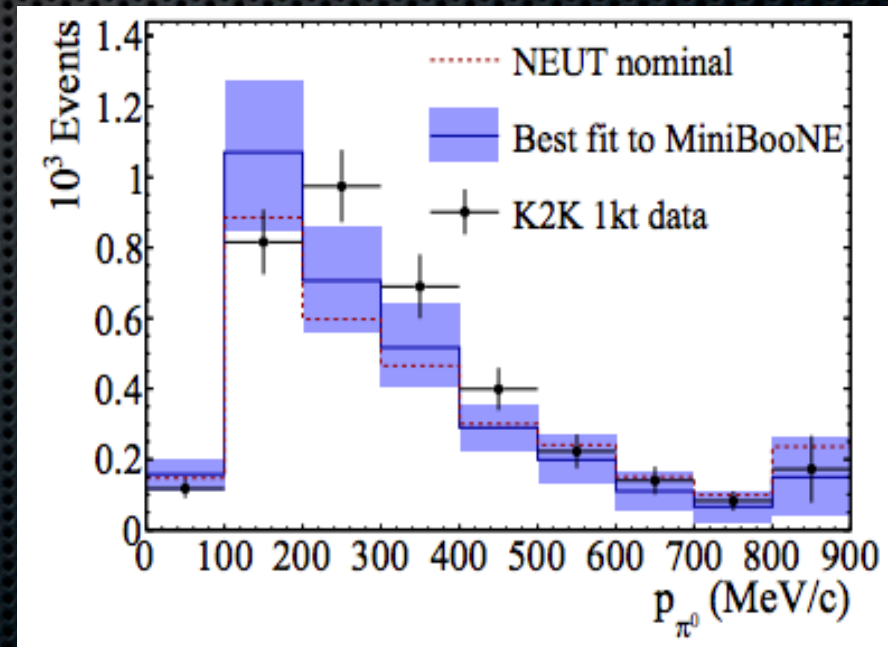
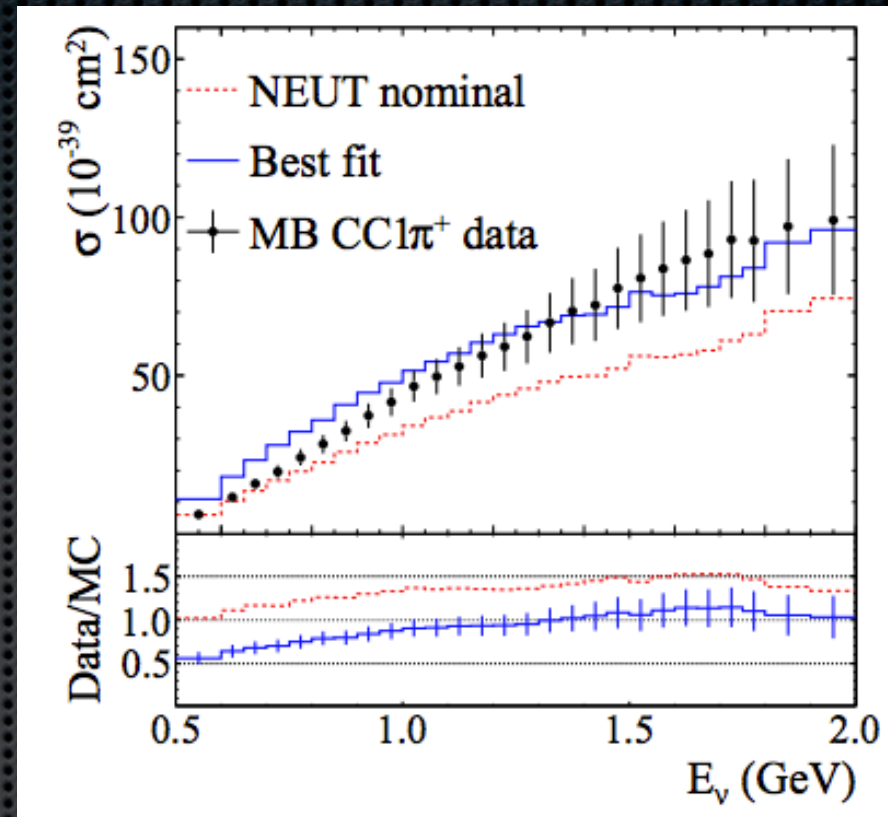
- ✦ Cross section uncertainties set by external data at  $\sim 1$  GeV from Mini-BooNE, other experiments.
- ✦ T2K primary neutrino interaction model is NEUT, with GENIE used as a cross-check.
- ✦ Signal
  - ✦ CCQE interactions use the model of Llewellyn Smith with nuclear effects described by relativistic Fermi gas model.
- ✦ Differences between NEUT and Mini-BooNE best fit used as prior uncertainty. ND280 further constrains models.





## Neutrino Cross Section Uncertainties

- ✦ Backgrounds
  - ✦ Single Pion Production CC1 $\pi$  main background for  $\nu_\mu$  disappearance: MisID'd as CCQE if pion is not identified
  - ✦ Pion production via hadronic resonances using Rein and Seghal Model
  - ✦ NC $\pi^0$  backgrounds main background to  $\nu_e$  appearance, flux dependant and can mimic a CC  $\nu_e$  interaction
  - ✦ Results from Mini-BooNE NC $\pi^0$  fit compared with K2K data (same target material as SK)

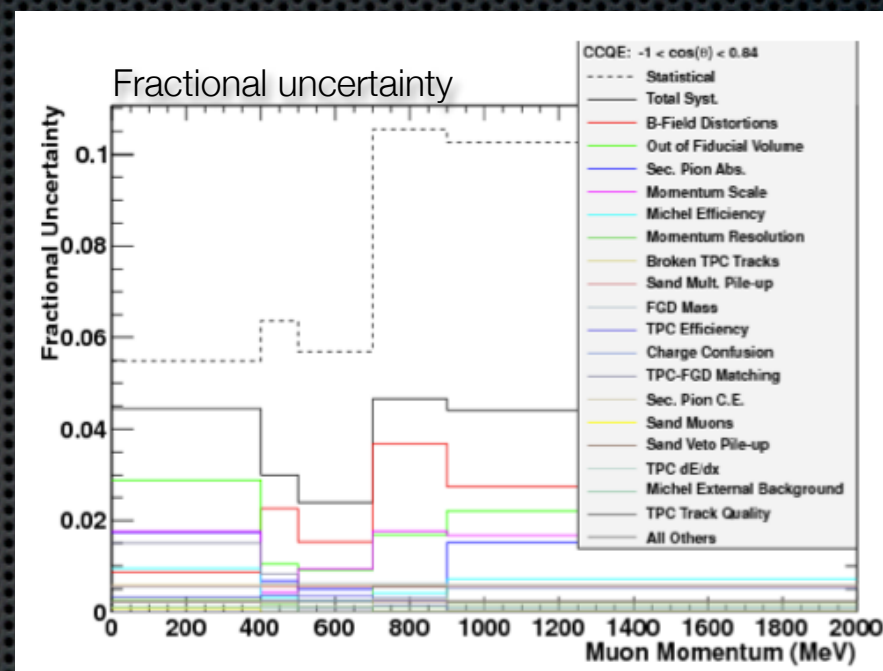




# ND280 $\nu_\mu$ measurements in CCQE and CCnonQE samples

## Systematics

- ✦ Statistics limited analysis
- ✦ Major Systematics
  - ✦ Magnetic field distortions in TPCs
  - ✦ background from interaction outside the FGD
  - ✦ Secondary pion interactions
- ✦ Uncertainty given in terms of  $p$ - $\theta$  bins  
40x40 covariance for each systematic



Systematic error	Error Size (%)	
	Minimum and maximum fractional error	Total fractional error
B-Field Distortions	0.3 - 6.9	0.3
Momentum Scale	0.1 - 2.1	0.1
Out of FV	0 - 8.9	1.6
Pion Interactions	0.5 - 4.7	0.5
All Others	1.2 - 3.4	0.4
Total	2.1 - 9.7	2.5



## SK Detector/Selection Uncertainties

- SK DAQ timing cuts.
- Event is fully contained in inner detector Reconstructed vertex is within fiducial volume
- Only one reconstructed ring.

### $\nu_e$ Selection

- Ring is electron like
- Visible energy is greater than 100 MeV
- No Michel electron
- Invariant mass is not consistent with  $\pi^0$  mass
- Reconstructed energy is less than 1250 MeV

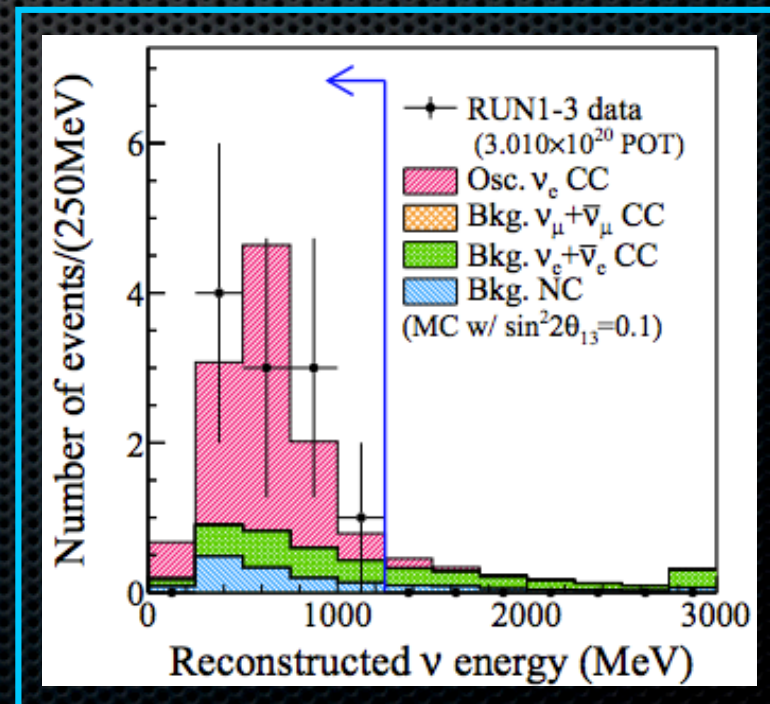
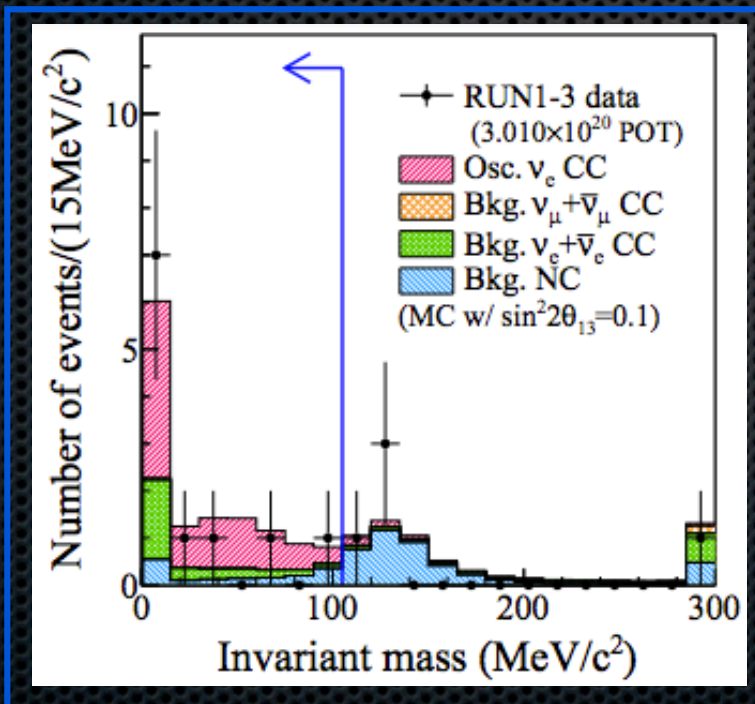
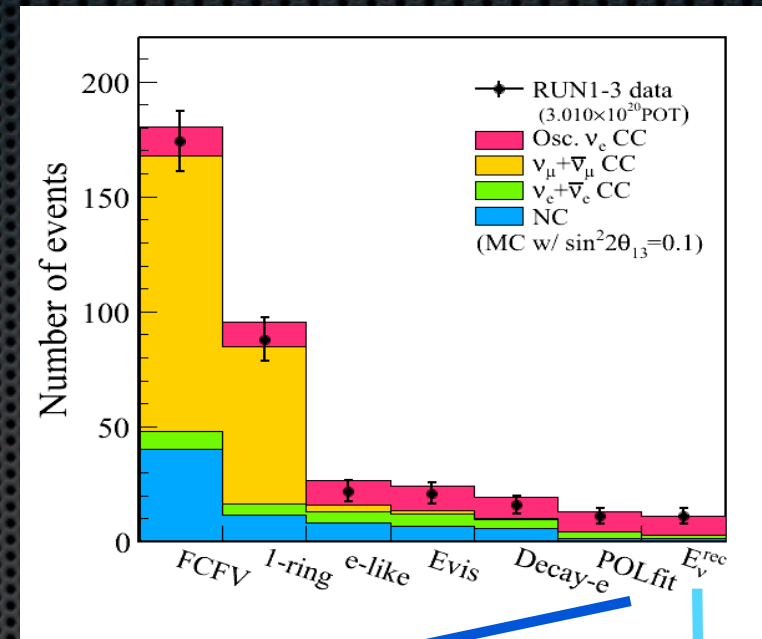
### $\nu_\mu$ Selection

- Ring is muon like
- Reconstructed muon momentum is greater than 200 MeV.
- 1 or less Michel electron



# $\nu_e$ Selection

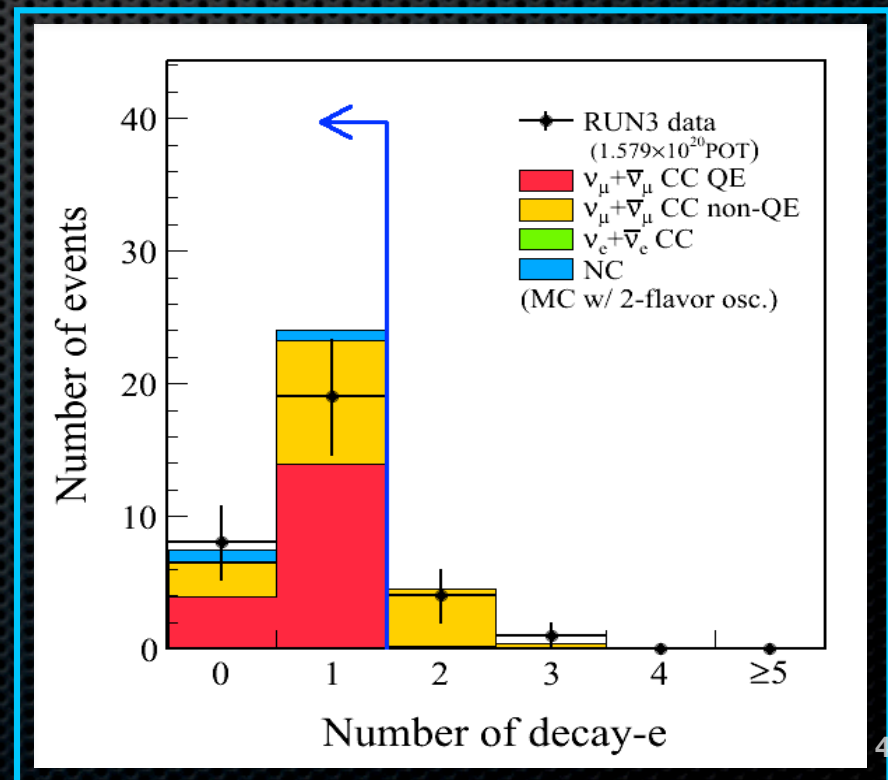
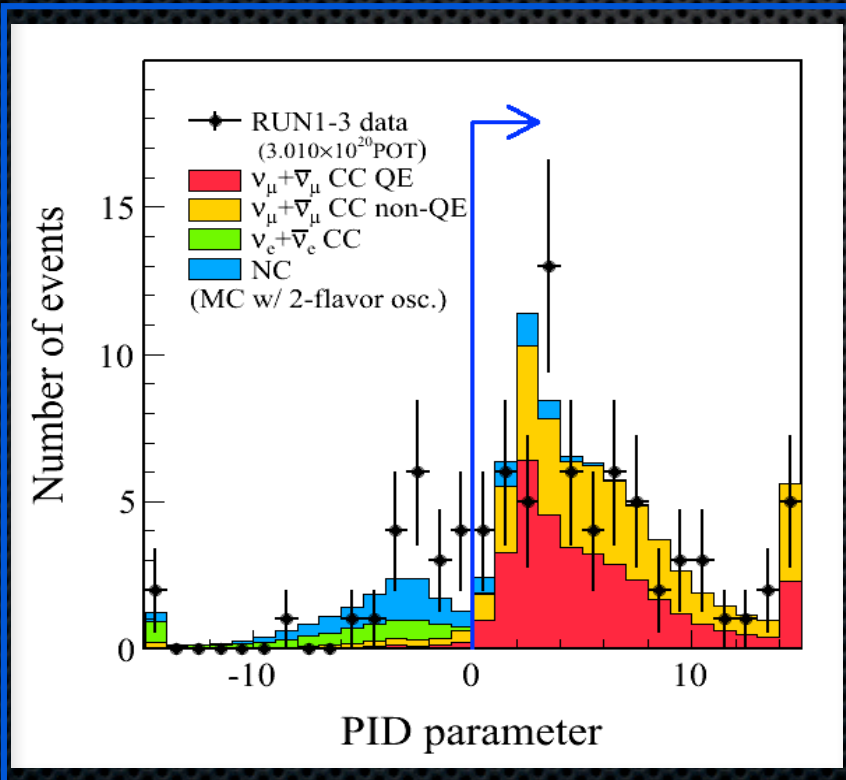
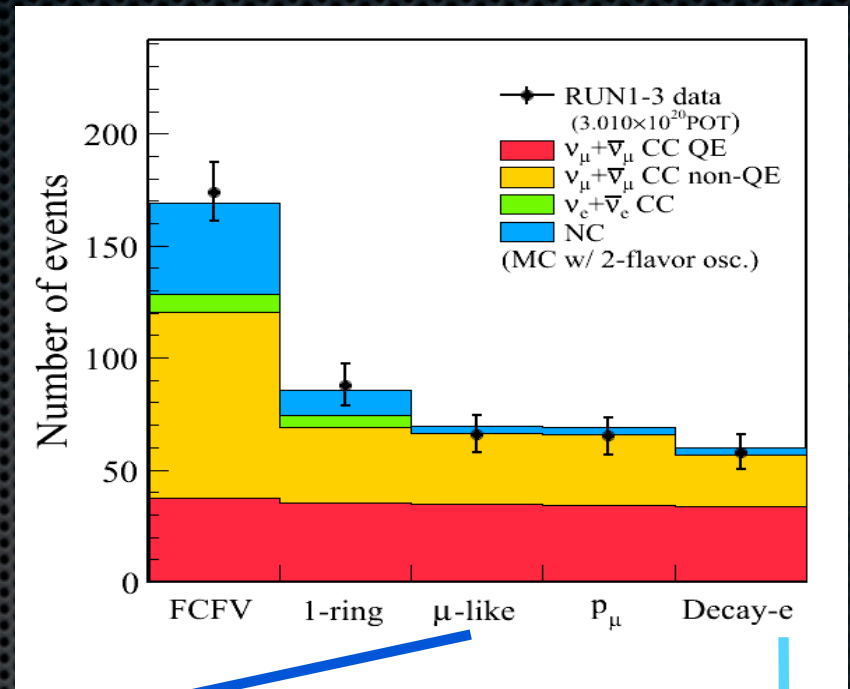
- Ring is electron like
- Visible energy is greater than 100 MeV
- No Michel electron
- Invariant mass is not consistent with  $\pi^0$  mass
- Reconstructed energy is less than 1250 MeV





# $\nu_\mu$ Selection

- Ring is muon like
- Reconstructed muon momentum is greater than 200 MeV.
- 1 or less Michel electron

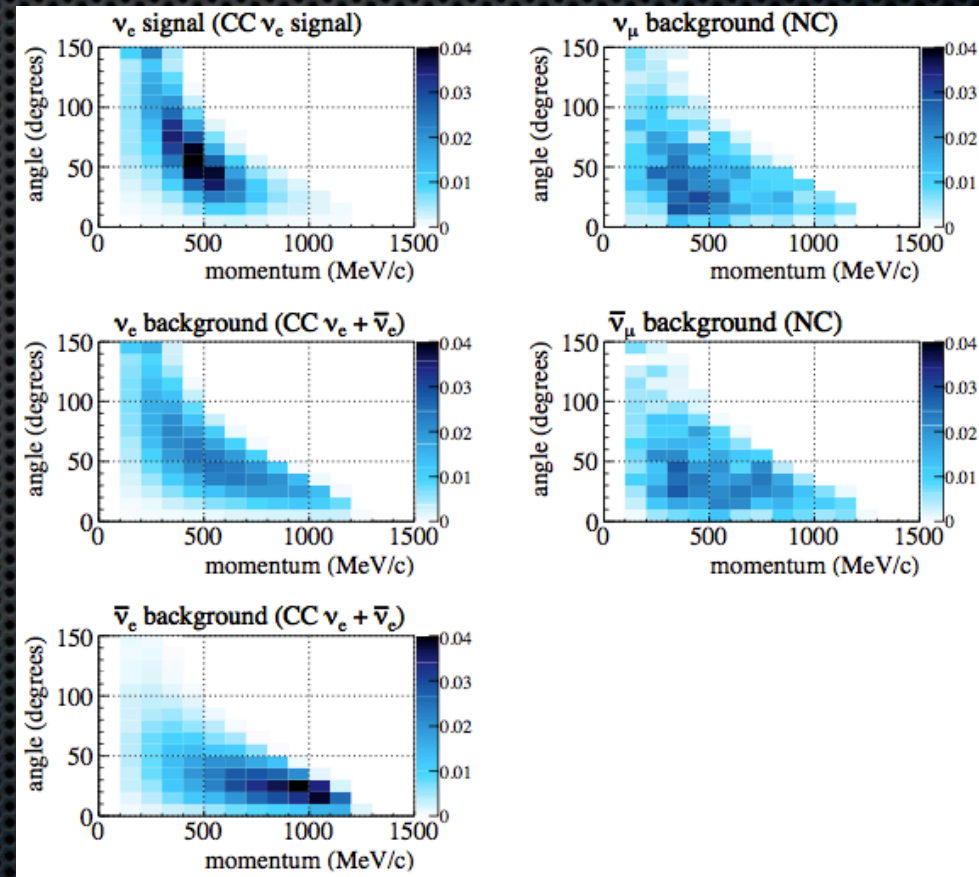




# $\nu_e$ appearance oscillation fit

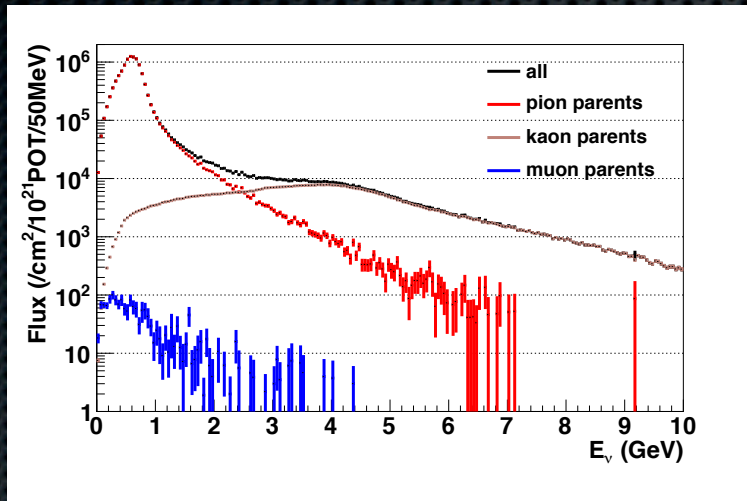
$$\mathcal{L}(N_{obs}, x; o, f) = \mathcal{L}_{norm}(N_{obs}; o, f) \times \mathcal{L}_{shape}(x; o, f) \times \mathcal{L}_{sys}(f)$$

- Extended likelihood fit of the reconstructed electron momentum and angle spectrum
  - x: measurements in  $(p_e, \theta_e)$
  - o: oscillation parameters
  - f: systematic parameters
- Fit templates of  $\nu_e$  signal and background.
  - Backgrounds have a wider range in kinematic space.

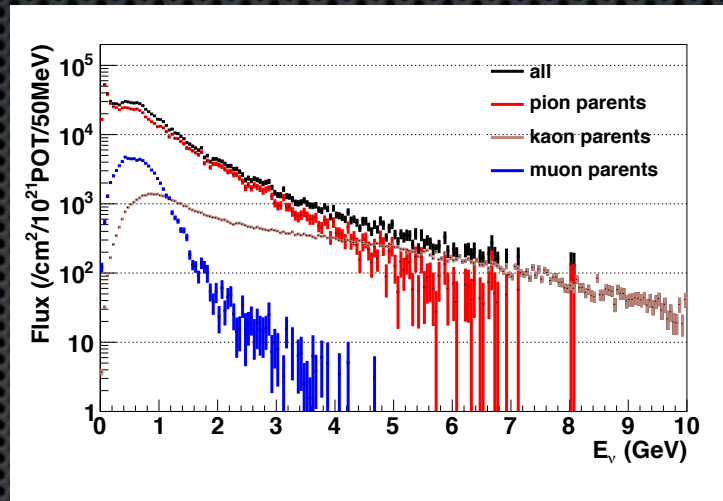




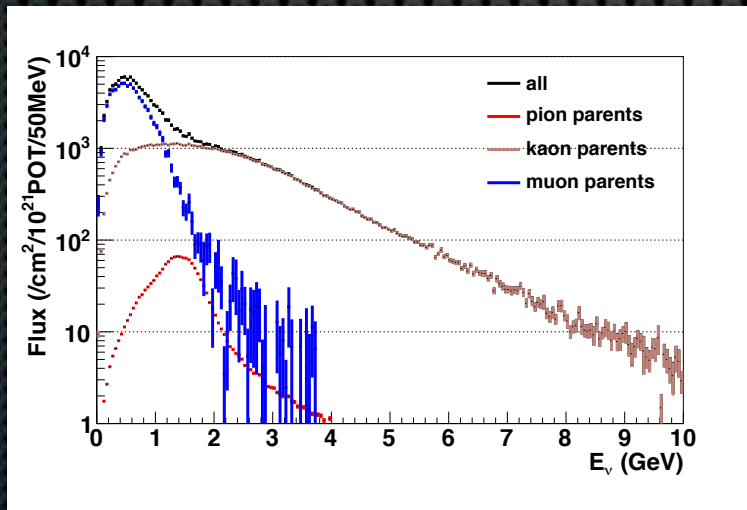
# SK Flux



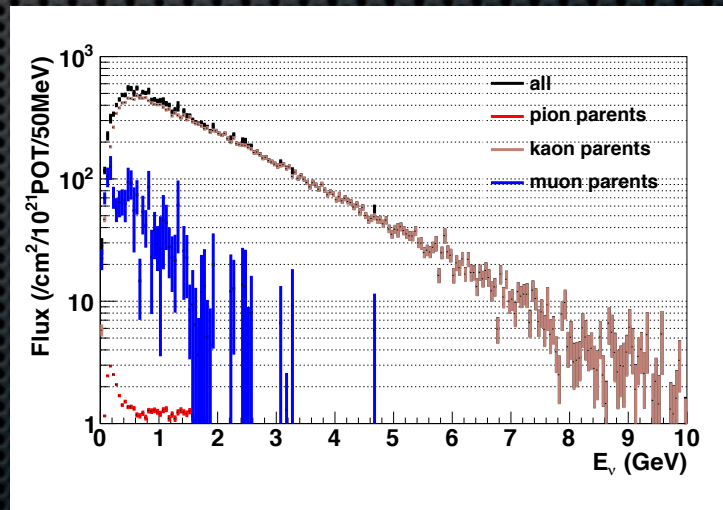
(a)  $\nu_\mu$  flux



(b)  $\bar{\nu}_\mu$  flux



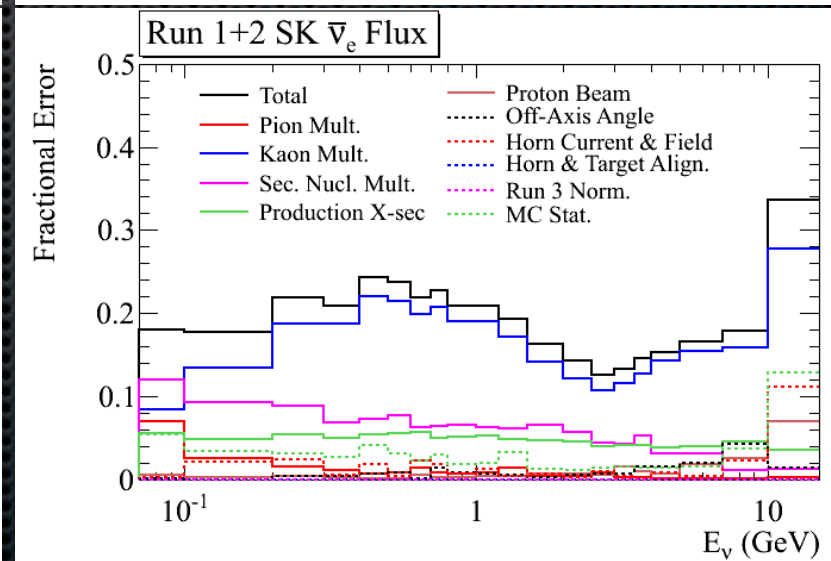
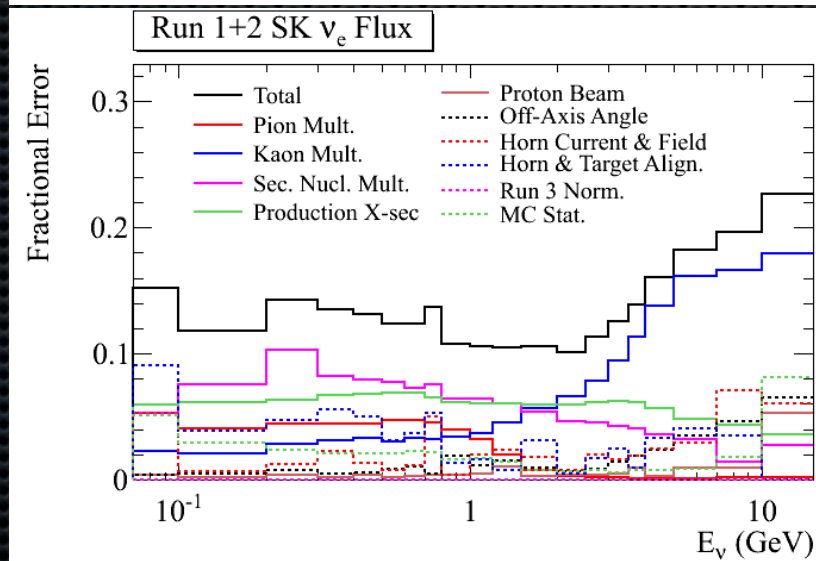
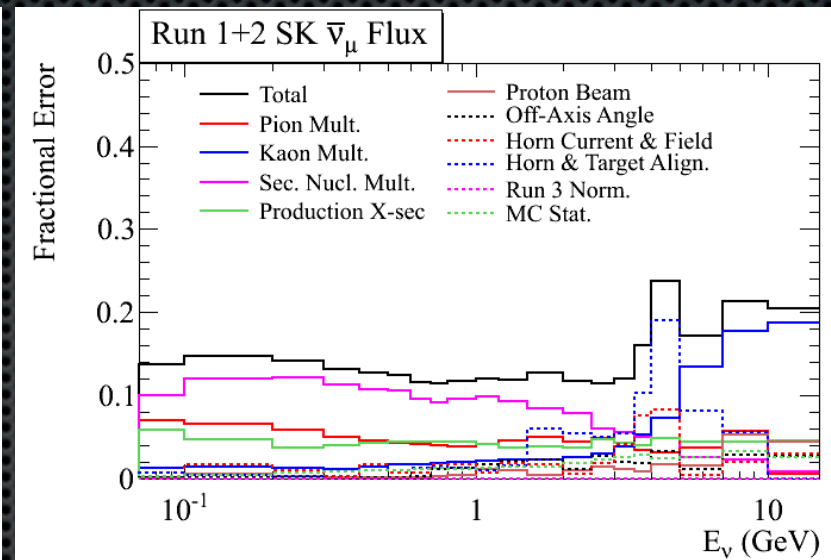
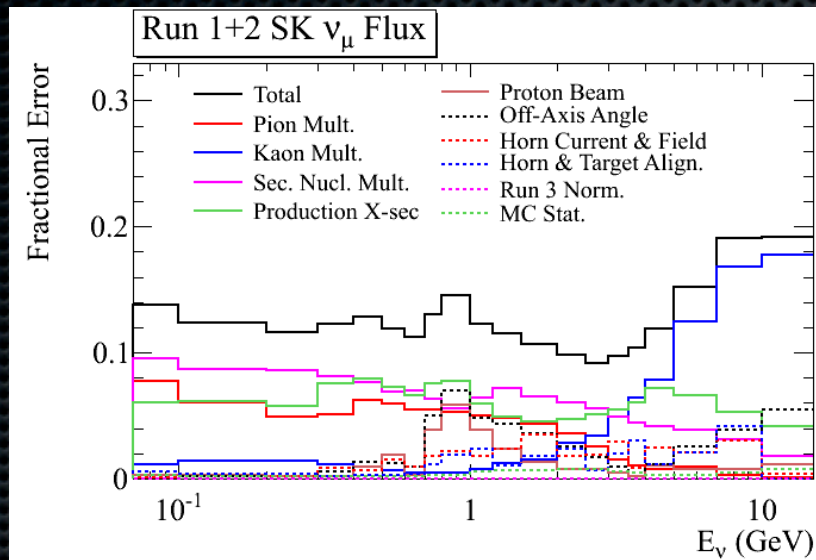
(c)  $\nu_e$  flux



(d)  $\bar{\nu}_e$  flux



# SK Flux Uncertainties.





# Beam Uncertainties

$$\sin^2(2\theta_{13})=0.1 \quad \Delta m_{32}^2=2.4 \times 10^{-3} \text{ eV}^2$$

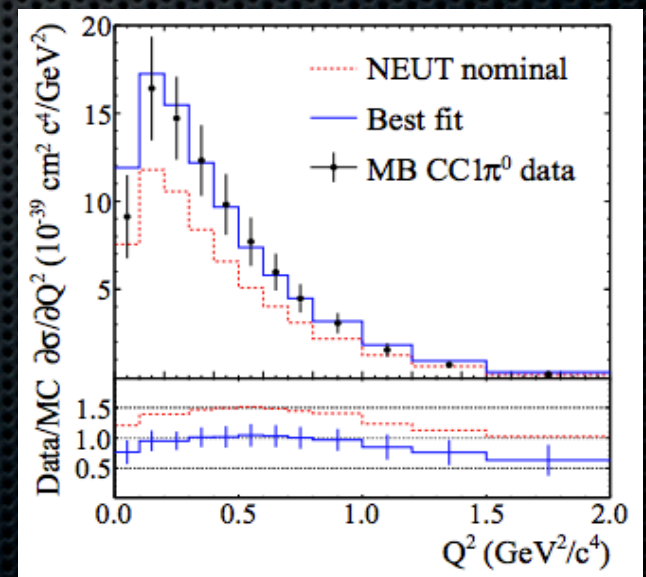
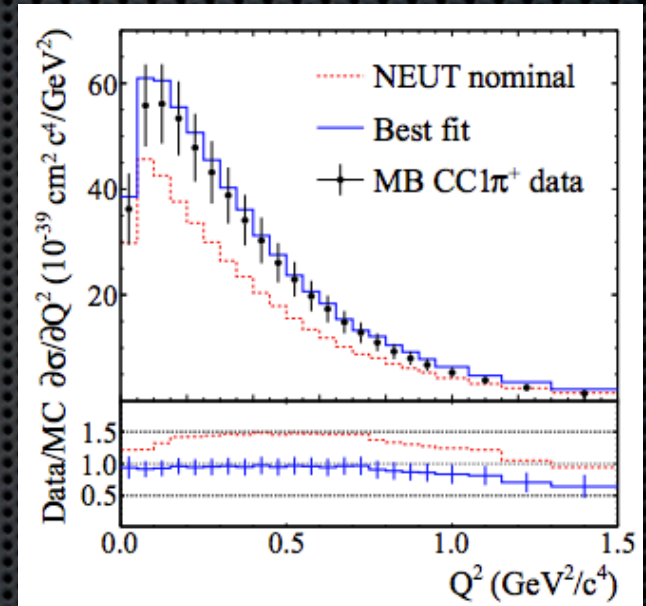
$$\sin^2(2\theta_{23})=1.0$$

	% Errors on Sample Predictions		
	$N_{ND}$	$N_{SK}$	$N_{SK}/N_{ND}$
Pion Production	3.41	4.97	1.88
Kaon Production	3.48	1.17	2.99
Secondary Nucleon Production	5.46	6.61	1.34
Hadronic Interaction Length	5.78	6.56	1.90
Proton Beam, Alignment & Off-axis Angle	3.45	2.08	1.75
Horn Current and Magnetic Field	1.40	1.16	1.39
Total	10.04	10.94	4.78



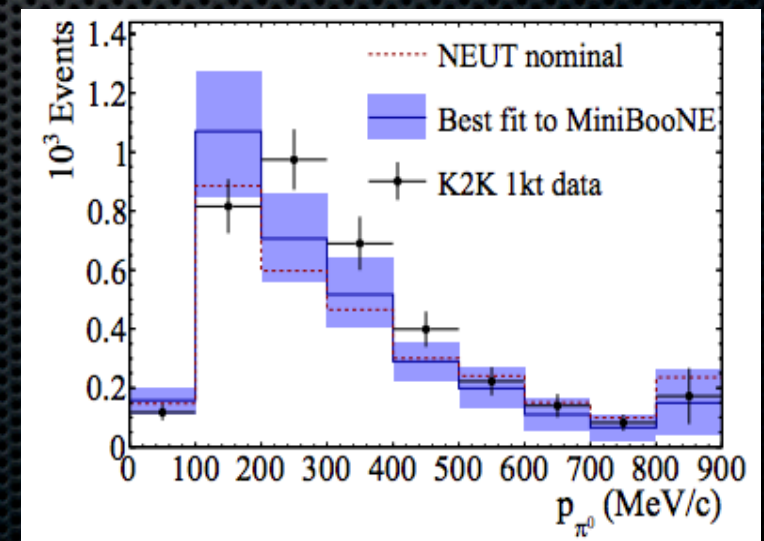
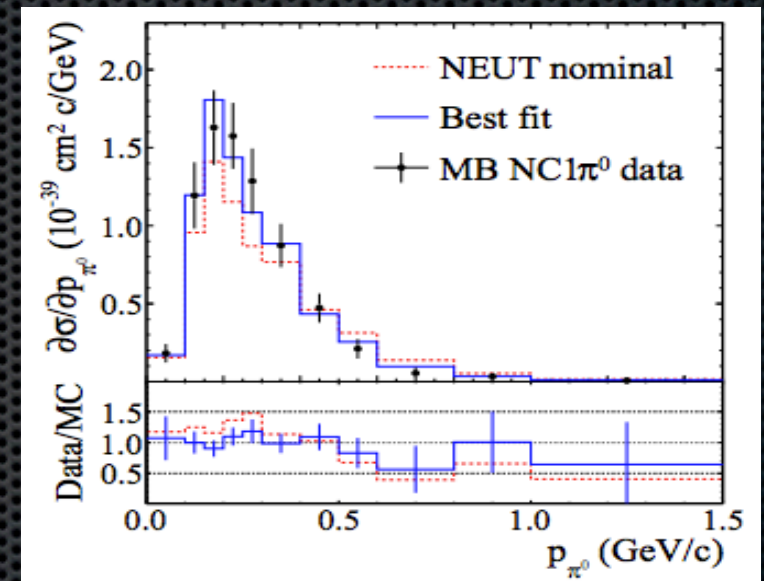
# CCQE backgrounds

- Joint Fit is done on Mini-BooNE CC1 $\pi^+$  CC1 $\pi^0$  and NC1 $\pi^0$  data.
- CC1 $\pi$  Single Pion Production CC1 $\pi$
- Main Background for  $\nu\mu$  disappearance: Same as CCQE if pion is not identified
- Pion production via hadronic resonances using Rein and Seghal Model. Uses axial mass MARES and several normalization parameters.
- Parameters MARES, CC1 $\pi$  and NC1 $\pi^0$  are propagated to ND280.





- ✧  $\nu_e$  appearance: NC backgrounds are flux dependant and can mimic a CC  $\nu_e$  interaction
- ✧ Results from Mini-BooNE NC fit compared with K2K data.
- ✧ K2K same nuclear target as SK.

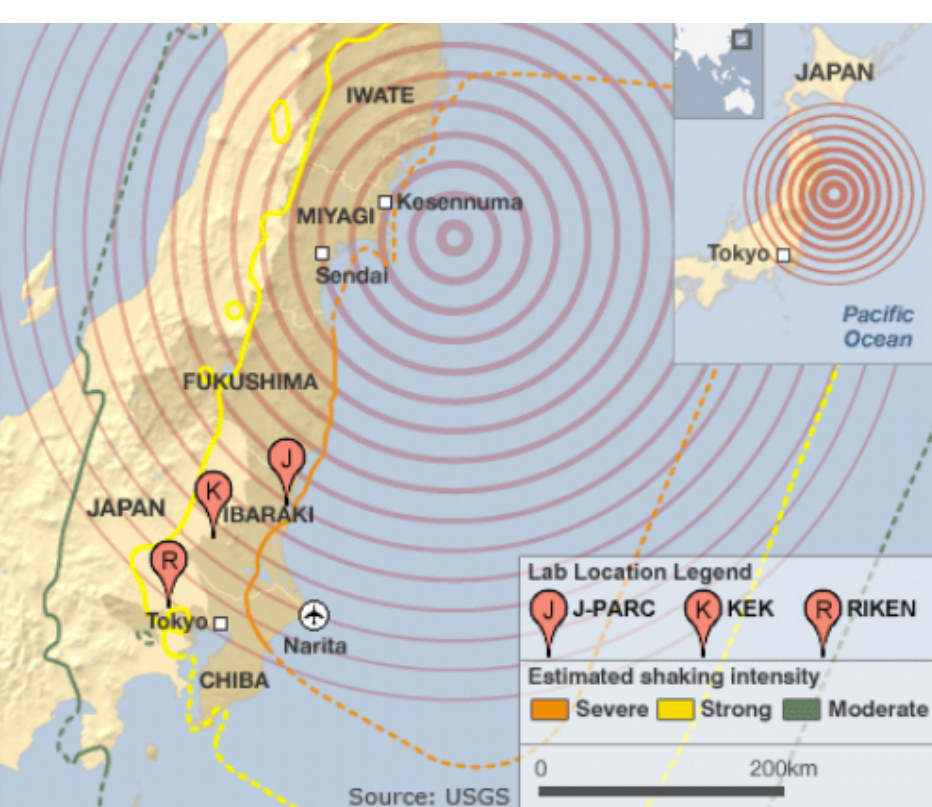




## Systematic error contribution to the predicted number of events in oscillation analysis (%)

Error source	$\sin^2 2\theta_{13} = 0$		$\sin^2 2\theta_{13} = 0.1$	
	w/o ND280 fit	w/ ND280 fit	w/o ND280 fit	w/ ND280 fit
Beam only	10.8	7.9	11.8	8.5
$M_A^{QE}$	10.6	4.5	18.7	7.9
$M_A^{RES}$	4.7	4.3	2.3	2.0
CCQE norm. ( $E_\nu < 1.5$ GeV)	4.6	3.7	7.8	6.2
CC1 $\pi$ norm. ( $E_\nu < 2.5$ GeV)	5.3	3.7	5.5	3.9
NC1 $\pi^0$ norm.	8.1	7.7	2.4	2.3
CC other shape	0.2	0.2	0.1	0.1
Spectral Function	3.1	3.1	5.4	5.4
$p_F$	0.3	0.3	0.1	0.1
CC coh. norm.	0.2	0.2	0.2	0.2
NC coh. norm.	2.1	2.1	0.6	0.6
NC other norm.	2.6	2.6	0.8	0.8
$\sigma_{\nu_e}/\sigma_{\nu_\mu}$	1.8	1.8	2.6	2.6
W shape	2.0	2.0	0.9	0.9
pion-less $\Delta$ decay	0.5	0.5	3.5	3.5
CC1 $\pi$ , NC1 $\pi^0$ energy shape	2.5	2.5	2.2	2.2
SK detector eff.	7.1	7.1	3.1	3.1
FSI	3.1	3.1	2.4	2.4
SK momentum scale	0.0	0.0	0.0	0.0
Total	21.5	13.4	25.9	10.3





- ✦ T2K/J-PARC has recovered from the “Great East Japan Earthquake” March 2011.
- ✦ Dec 9th LINAC operation restarted.
- ✦ Dec 24th. Neutrino events observed in T2K-ND80.





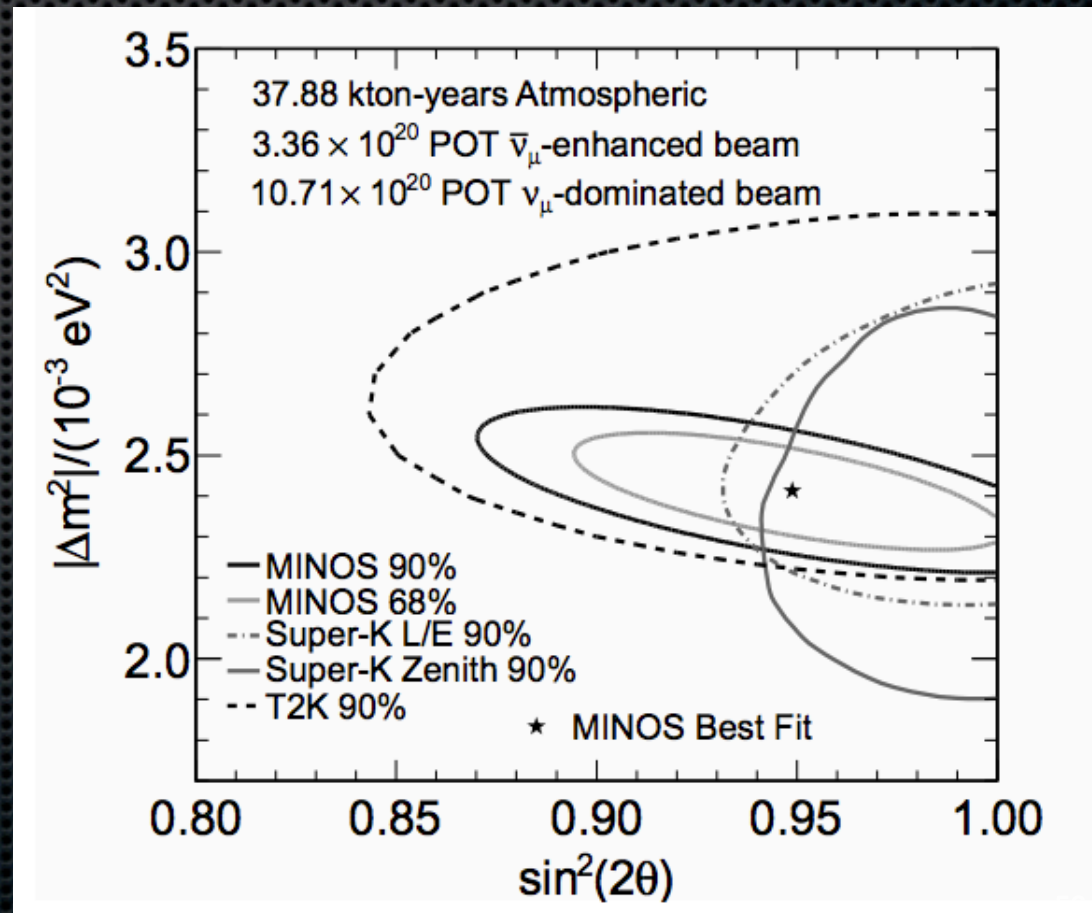
# Markov Chain Monte Carlo

- ✦ Sample from a multidimensional probability distribution is a with a directed random walk.
  - ✦ Randomly move from one point to another in your multidimensional space.
  - ✦ If the probability density is higher the second point, step to that point, if it is lower accept with a probablilty  $P = P(\text{current})/P(\text{proposed})$



# MINOS

- ✦ Latest results measure non maximal  $\theta_{23}$
- ✦ arXiv:1304.6335



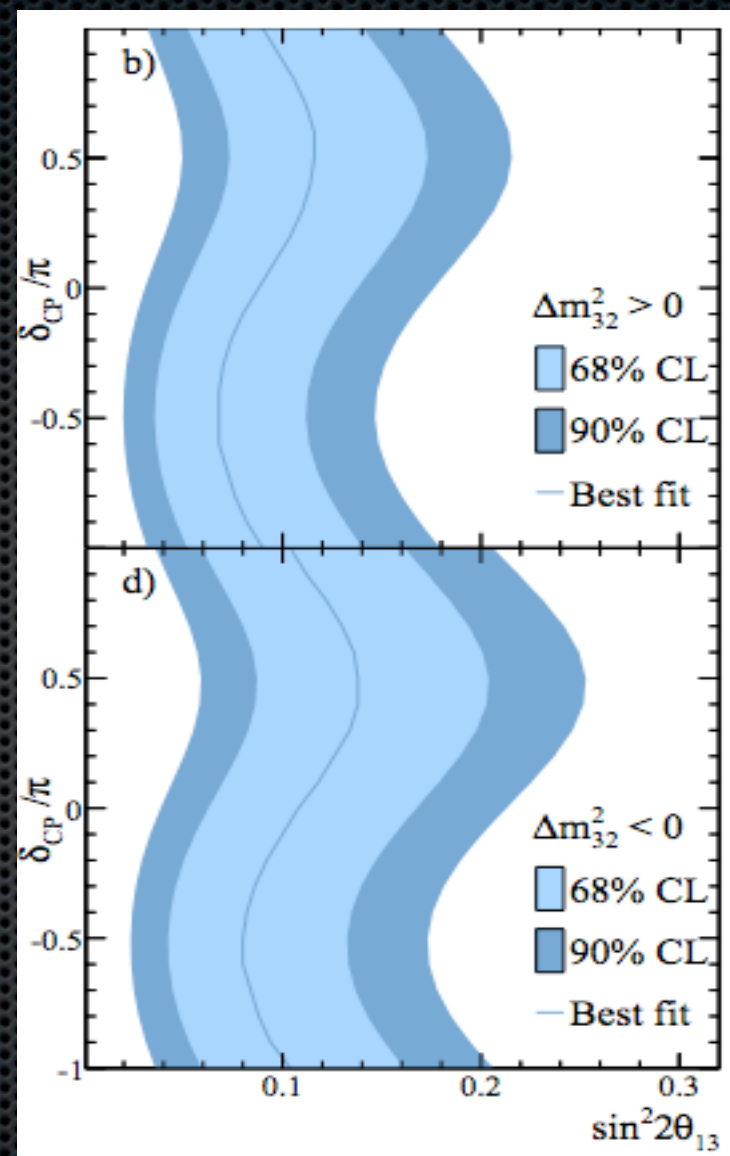
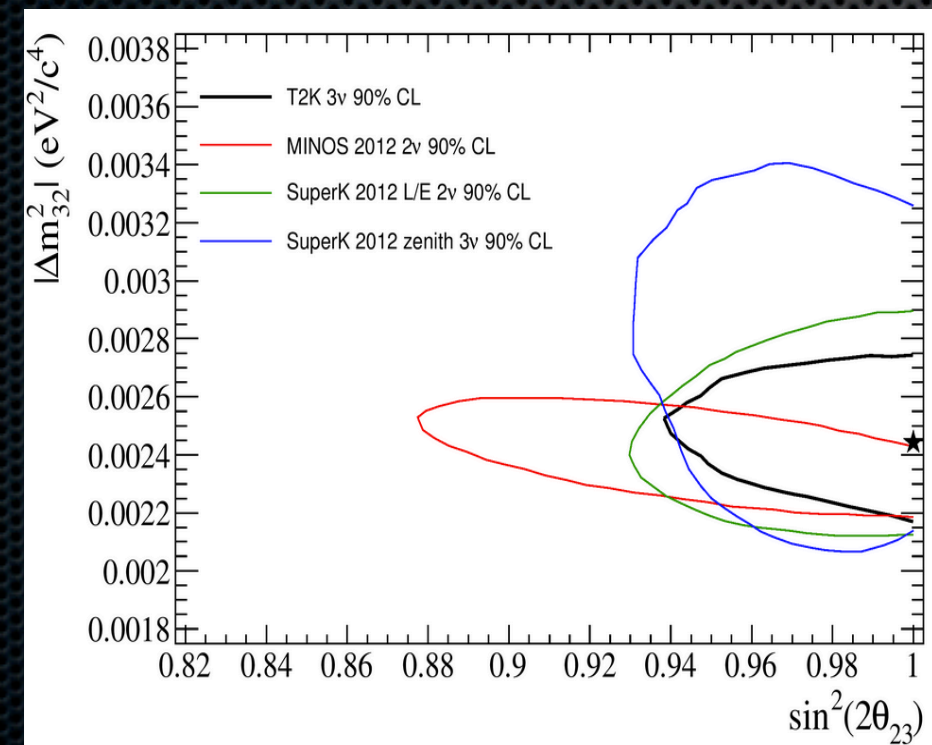


# Measurements

90% CL

$$2.14 \times 10^{-3} \text{eV}^2 < |\Delta m_{32}^2| < 2.76 \times 10^{-3} \text{eV}^2$$

$$\sin^2 2\theta_{23} > 0.957$$



$$\sin^2 2\theta_{13} = 0.088_{-0.039}^{+0.049} \text{ (normal hierarchy)}$$

$$\sin^2 2\theta_{13} = 0.108_{-0.046}^{+0.059} \text{ (inverted hierarchy)}$$